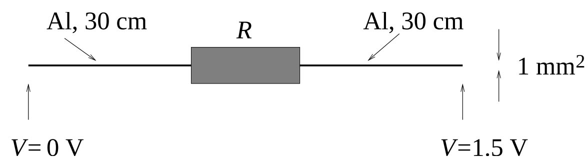
Institutt for fysikk, NTNU
TFY4155/FY1003: Elektrisitet og magnetisme
Vår 2005

Øving 10

Guidance: Thursday March 17 and Friday March 18
To be delivered by: Wednesday March 30

Exercise 1

A voltage source $V = 1.5$ V is connected to a resistor with resistance $R = 10\ \Omega$ via two 30 cm long aluminum wires with cross section 1 mm^2.



a) How big is the voltage drop over the Al wires and the resistor, respectively? [Answers: 2.5 mV and 1.497 V]

b) Determine the current and the power dissipated ("lost") in the resistor. [Answers: ca 0.15 A and 0.23 W]

c) What is the average drift velocity of the free electrons through the Al wires? Assume one free electron from each Al atom. Compare with the average thermal velocity for an electron at room temperature. (Average kinetic energy pr electron at temperature T is $3k_BT/2$, where k_B is Boltzmann's constant.)
[Answers: 15.6 μm/s and ca 10^5 m/s.]

Given information: Mass density of Al: 2700 kg/m^3. Molar mass of Al: 26.98 g/mol. Electric conductivity for Al at room temperature: $3.54 \cdot 10^7\ \Omega^{-1}\text{m}^{-1}$. Boltzmann's constant: $k_B = 1.38 \cdot 10^{-23}$ J/K.

Exercise 2

An electronic flash contains a capacitor for storage of energy for the flash of light. When the flash is triggered, the stored charge on the capacitor is quickly discharged and the energy is lost in an electric discharge in a gas-filled tube. Assume we have a flash where the duration of the light is 5 ms, with an average effect of 700 W.

a) If the efficiency is 90% in the transformation of electric energy into light energy (the remaining 10% is transferred into heat), how much energy must be stored in the capacitor for one "blink"? [Answer: 3.89 J]

b) If the capacitor has a capacitance 0.80 mF, what is the potential difference required between the capacitor plates in order to store the required energy? [Answer: 98.6 V]

Exercise 3

A current runs along a straight, cylindrical conductor with circular cross section (radius R). The current density is largest at the center of the conductor and decays with the distance r from the center in the following manner:

$$j(r) = j_0 \left(1 - \frac{r^2}{R^2}\right)$$

Thus, at the surface of the conductor ($r = R$), we have zero current density. (The direction of $\boldsymbol{j}(r)$ is *along* the conductor.)
Show that the total current in the conductor is

$$I = \int \boldsymbol{j} \cdot d\boldsymbol{A} = \int j \cdot dA = \frac{1}{2} j_0 \pi R^2$$

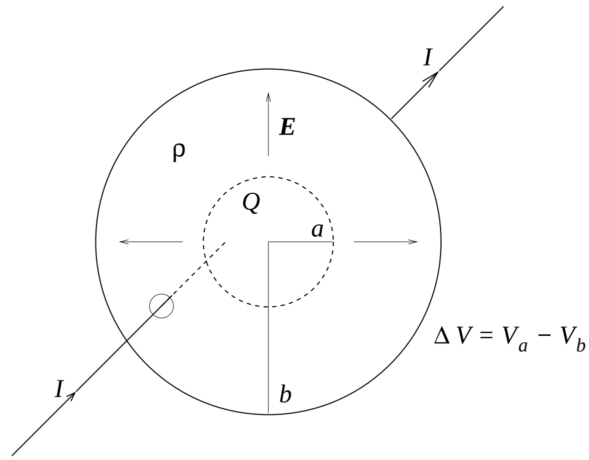
Hint: Start with the current dI running in a cylindrical "tube" with inner radius r, outer radius $r + dr$, and therefore cross section with area $dA = 2\pi r\, dr$.

Exercise 4

The figure shows two spherical conductors with radius a (the inner sphere) and b (the outer sphere), respectively. The region between the conducting spheres is filled with a material with resistivity ρ.
(Note: Here, the symbol ρ denotes resistivity, or inverse conductivity, since $\rho = 1/\sigma$. So here, ρ does not mean charge pr unit volumd...!)
A thin, isolated conducting wire passes through a little hole in the outer spherical conductor and in to the inner sphere. A stationary (i.e., time independent) electric current runs "through the system" as shown in the figure. Then, the potential difference between the inner and outer sphere is $\Delta V = V_a - V_b$, with the largest value of the potential on the inner sphere. Assume that the connecting wires has a negligible resistance in comparison with the material between the two spheres. Show that the resistance of this system is $R = \rho(a^{-1} - b^{-1})/4\pi$. You may do this in one of two ways (or both, if you like!):

1. Start with the resistance of a thin spherical shell with radius r and thickness dr, which is $dR = \rho dr/4\pi r^2$.

2. Start by assuming that the inner sphere has charge Q, and determine the two quantities ΔV and the current $I = \int \boldsymbol{j} \cdot d\boldsymbol{A} = \rho^{-1} \int \boldsymbol{E} \cdot d\boldsymbol{A}$. (So you may use Gauss' law...!)


I
E
ρ
Q
a
b
I
ΔV = V_a − V_b