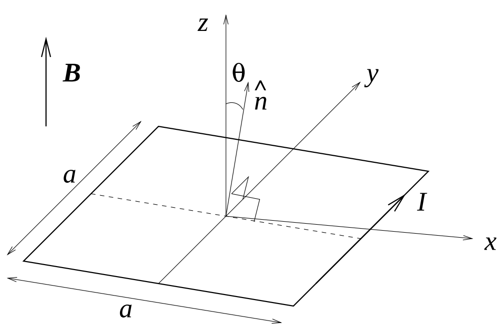
Institutt for fysikk, NTNU
TFY4155/FY1003: Elektrisitet og magnetisme
Vår 2005

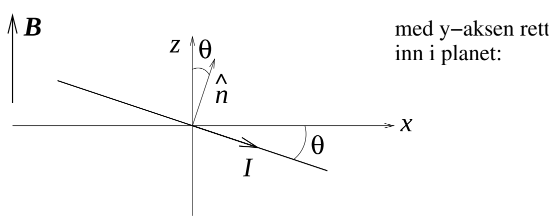
Øving 13

Guidance: Thursday April 14 and Friday April 15
To be delivered by: Monday April 18

Exercise 1

In the lectures, we showed that atoms may be viewed as small current loops, i.e., as small magnetic dipoles with magnetic dipole moment $\boldsymbol{m} = I\boldsymbol{A}$, where the current I goes in a loop which encloses a (planar) area A. ("The vector area" is then $\boldsymbol{A} = A\,\hat{n}$, where $\hat{n}$ is a unit vector perpendicular to the enclosed surface, with the positive direction determined by the right hand rule.)
Here, we will use a *quadratic* current loop as a model for such an atomic magnetic dipole and look closer at how it will behave in a magnetic field $\boldsymbol{B}$. (We could have used a circular loop, but the quadratic one is a little simpler when it comes to the calculations...)
The current loop has edges with length a and transports a current I. It is placed in a *homogeneous* magnetic field $\boldsymbol{B} = B\,\hat{z}$ and is allowed to rotate freely around the y-axis, which in our case passes through the centre of the current loop, as shown in the figure:





The orientation of the current loop is defined through the angle θ between the z-axis and the surface normal $\hat{n}$. (Positiveθ *counterclockwise*, as shown in the figure.)

a) What is the magnetic dipole moment $\boldsymbol{m}$ of this current loop? What is the total force due to $\boldsymbol{B}$ on the current loop?

b) Find the torque $\boldsymbol{\tau}$ on the loop with respect to the y-axis and show that it can be written in the form $\boldsymbol{\tau} = \boldsymbol{m} \times \boldsymbol{B}$.

[Hint: Find the force on each of the four straight elements of the loop and use the face that torque equals "arm times force".]

c) Determine the potential energy $U(\theta)$ of such a magnetic dipole in the field $\boldsymbol{B}$. Draw a sketch of $U(\theta)$. Which orientation of the dipole with respect to $\boldsymbol{B}$ represents a stable and an unstable equilibrium, respectively?

d) In iron, each atom has a magnetic dipole moment $\boldsymbol{m}_{\text{Fe}}$ which is made up of two parallel electron spins, so that $m_{\text{Fe}} = 2\mu_B$. Here, $\mu_B = e\hbar/2m_e$ is the magnetic dipole moment of a single electron spin, the so-called Bohr magneton, which has the value $9.27 \cdot 10^{-24}$ Am^2.
What is then the maximum density of magnetic dipole moment, i.e., the maximum magnetic dipole moment pr unit volume, in iron?
[Comment: Magnetic dipole moment pr unit volume is, by definition, the quantity *magnetization*. In electrostatics, we introduced *polarization*, which by definition is electric dipole moment pr unit volume. More about magnetism and magnetization in the lectures!]

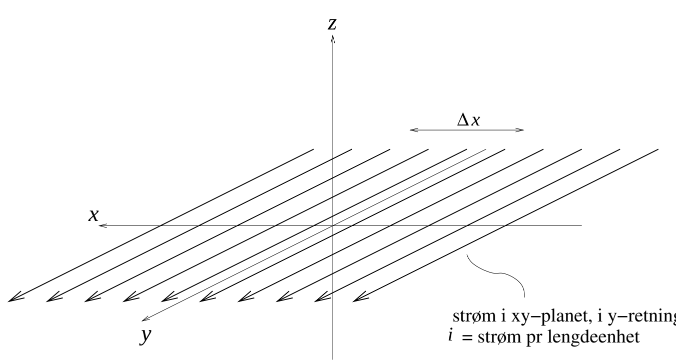
Given information: Molar mass of iron: 55.9 g/mol. Mass density of iron: 7.9 g/cm^3. 1 mol = $6.02 \cdot 10^{23}$.

Exercise 2

Show, by using Ampere's law, that the magnetic field $\boldsymbol{B}$ from a uniform "surface current" $\boldsymbol{i} = i\,\hat{y}$ flowing in the (complete) xy-plane in the positive y direction is

$$\boldsymbol{B} = \begin{cases} -(\mu_0 i/2)\,\hat{x} & \text{for } z < 0 \\ +(\mu_0 i/2)\,\hat{x} & \text{for } z > 0 \end{cases}$$

(I.e., independent of the distance from the xy plane, just like we found for the electric field from an infinitely large uniformly charged plane.) Here, i is the current *pr unit length* of the x direction. In other words, in a "stripe" of width Δx runs a current $\Delta I = i \cdot \Delta x$.



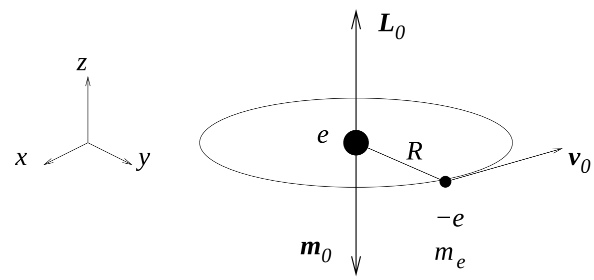
Hint:

- You have already been informed that the y and the z component of $\boldsymbol{B}$ are both zero. However, spend some time to convince yourself that it has to be like that! Such an "investigation" of the symmetry of the problem is completely *essential* if you want to take advantage of Ampere's law in order to determine the magnetic field. Often, you then need to go back to the Biot-Savart law and look at the consequences of "current elements" $I\, d\boldsymbol{l}$ that give contributions $d\boldsymbol{B} \sim I\, d\boldsymbol{l} \times \hat{r}$ to the total magnetic field.

- In this particular problem, you will perhaps convince yourself that a sensible choice of "amperian loop" is a rectangular curve with surface normal in the current direction. If so, you are on the right track!

Exercise 3

In this exercise, we shall use a classical model of an atom and take a closer look at how an external magnetic field $\boldsymbol{B}$ will influence the orbital movement of the electron around the nucleus. Such a *diamagnetic response* is present in all atoms. (More about different types of magnetism in the lectures.) Let's for simplicity have a hydrogen atom in our thoughts, with a single electron with charge $-e$ in a circular orbit (in the xy plane) with radius R around a nucleus with charge $+e$.

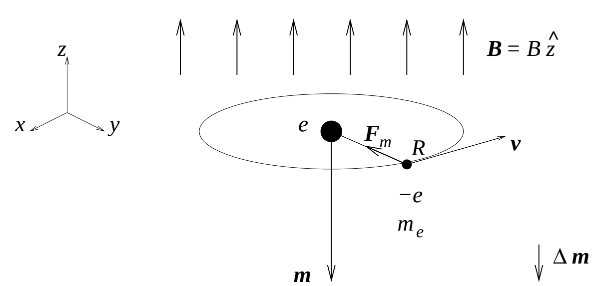


a) Without an external magnetic field, the velocity of the electron is v_0. Show that uniform circular motion in the Coulomb field from the nucleus then results in a circular orbit with radius

$$R = \frac{e^2}{4\pi\varepsilon_0 m_e v_0^2}$$

What is the angular momentum $\boldsymbol{L}_0$ and the magnetic dipole moment $\boldsymbol{m}_0$ of this electron? (Here, we ignore the spin of the electron.)

b) Next, we turn on a magnetic field $\boldsymbol{B}$, for simplicity with direction perpendicular to the circular orbit of the electron.



Now, the electron will be affected by an additional force (i.e., in addition to the Coulomb force from the proton in the nucleus), namely the magnetic force $\boldsymbol{F}_m = -e\boldsymbol{v} \times \boldsymbol{B}$ which of course must result in a modified equation of motion. Consequently, we must find a different relation between the velocity v of the electron and the radius of the circular orbit, R. Let us further assume that the magnetic field only changes the velocity (and not the radius R), and determine the new velocity v. Also find the new value $\boldsymbol{m}$ of the magnetic dipole moment of the electron and show that the *change*

$$\Delta\boldsymbol{m} = \boldsymbol{m} - \boldsymbol{m}_0$$

will always be directed *opposite to* the magnetic field $\boldsymbol{B}$. Show that this will be the case both for $\boldsymbol{B}$ "up" and "down" with respect to the original magnetic dipole moment $\boldsymbol{m}_0$ of the electron.

Comments:

1. We have earlier stated that a static magnetic field never does any work on a moving charge since $\boldsymbol{F}_m \perp \boldsymbol{v}$. Thus, a static magnetic field cannot change the magnitude of the velocity of the charge, apparently in conflict with what we have just found above. However, the point is that here we start with $B = 0$ and *turn on* a magnetic field. This means that we do not have a static magnetic field at all times, but rather a field that during a certain period of time must change from zero to its final value. As we will see later in the lectures, a time dependent magnetic field will create ("induce") an electric field. (This is Faraday's law of electromagnetic induction.) And, as you know, an electric field may very well change the velocity of an electron. This resolves the apparent paradox.

2. The sign of the diamagnetic response is actually an expression of *Lenz' law*, which you may have heard of earlier, and which we will come back to in the lectures: The "response" of the system to a change in the magnetic field is in a direction so that it attempts to reduce the effect of the applied change.

3. Strictly speaking, it is necessary with a *quantum mechanical* description in order to explain diamagnetism "properly". In fact, there is a *theorem* in statistical physics which states that for a system of classical charged particles in thermal equilibrium in an external magnetic field, the induced magnetic dipole moment is exactly zero (Bohr - van Leeuwen's theorem). In other words: Diamagnetism is actually a purely quantum mechanical effect! Still, the simple classical model described above, with one atom only, gives a useful qualitative picture of the effect.