Institutt for fysikk, NTNU
TFY4155/FY1003 Elektrisitet og magnetisme
Vår 2005

Solution to øving 10

Guidance Thursday March 17 and Friday March 18

Exercise 1

a) This system can be viewed as three resistances connected in series: the two 30 cm long Al wires and the resistance $R = 10\ \Omega$. The resistance of the two Al wires becomes

$$R_A = \frac{l}{\sigma A} = \cdot \frac{0.60\text{m}}{3.54 \cdot 10^7\ \Omega^{-1}\text{m}^{-1} \cdot 10^{-6}\text{m}^2} = 0.017\ \Omega$$

The same current I passes through the whole system. It is

$$I = \frac{V}{R + R_A} = \frac{1.5\ \text{V}}{10.017\ \Omega} = 0.1497\ \text{A} \simeq 0.15\ \text{A}$$

according to Ohm's law. Thus, we obtain the voltage drops

$$V_R = RI = 10\ \Omega \cdot 0.1497\ \text{A} = 1.497\ \text{V}$$

over the resistance R and

$$V_A = R_A I = 0.017\ \Omega \cdot 0.1497\ \text{A} = 0.0025\ \text{V}$$

over the two Al wires together.

b) We found the current I in *a*) above. The dissipated effect in the resistance R becomes

$$P = V_R I = 1.497\ \text{V} \cdot 0.1497\ \text{A} = 0.224\ \text{W}$$

(0.225 $\simeq$ 0.23 W if we neglect the resistance of and the voltage drop across the two Al wires)

c) Here, we must first find the density of free electrons n. Next, we may use $I = j \cdot A = nevA$ in order to calculate the mean drift velocity v.
In Al, we have a mass density 2700 kg pr m^3. This corresponds to 2700/0.02698 mol = 100074 mol = 100074 $\cdot 6.02 \cdot 10^{23}$ atomer = $6.02 \cdot 10^{28}$ atoms, and hence equally many free electrons, assuming one free electron pr Al atom. Mean drift velocity becomes

$$v = \frac{I}{neA} = \frac{0.15}{6.02 \cdot 10^{28} \cdot 1.6 \cdot 10^{-19} \cdot 10^{-6}} = 1.56 \cdot 10^{-5}\ \text{m/s} = 15.6\mu\text{m/s}$$

Average thermal velocity for the electrons may be estimated by setting the kinetic energy equal to the thermal energy:

$$\begin{aligned} \frac{1}{2}mv^2 &= \frac{3}{2}k_B T \\ \Rightarrow v &= \sqrt{\frac{3k_B T}{m}} \simeq 10^5\ \text{m/s} \end{aligned}$$

Here, $k_B = 1.38 \cdot 10^{-23}$ J/K is Boltzmann's constant. We see that the mean drift velocity is roughly 10 orders of magnitude smaller than the average thermal velocity. In other words, it takes several hours for a given electron to get from one end to the other in our system!

Exercise 2

a) If 700 W corresponds to 90% of the total effect, the total effect becomes 700/0.9 W = 778 W. This effect is supposed to be delivered during a period of 0.005 s. Then, an energy $(700/0.9) \cdot 0.005$ J = 3.89 J must be stored in the capacitor.

b) We have in the lectures derived an expression for the work required to charge a capacitor with capacitance C. Let us briefly remind ourselves: In order to increase the charge on the capacitor from q to $q + dq$, a work $dW = v(q)dq$ is required. Here, $v(q)$ is the voltage across the capacitor when the charge is q. Per definition, we have $C = q/v$, so that $dW = C^{-1}q\ dq$ and the total work must be $W = Q^2/2C$, i.e., in order to increase the charge from 0 to Q. With $Q = VC$, this can also be written as $W = CV^2/2$. This means that the voltage required to store an energy 3.89 J in a capacitor with capacitance 0.80 mF is

$$V = \sqrt{\frac{2W}{C}} = \sqrt{\frac{2 \cdot 3.89}{0.80 \cdot 10^{-3}}} = 98.6 \text{ V}$$

Exercise 3

Let us imagine we divide the conductor into cylindrical "tubes" with inner radius r, outer radius $r + dr$, and therefore cross sections with area

$$dA = 2\pi r\ dr$$

The current in such a tube is

$$dI = j \cdot dA = j_0 \left(1 - \frac{r^2}{R^2}\right) \cdot 2\pi r\ dr$$

The total current I is found by integrating dI over the full cross section of the conductor, i.e., by letting r vary from 0 to R:

$$\begin{aligned} I &= \int dI \\ &= \int_0^R j_0 \left(1 - \frac{r^2}{R^2}\right) \cdot 2\pi r\ dr \\ &= 2\pi j_0 |_0^R \left(\frac{1}{2} r^2 - \frac{r^4}{4R^2}\right) \\ &= 2\pi j_0 \left(\frac{1}{2} R^2 - \frac{1}{4} R^2\right) \\ &= \frac{1}{2} j_0 \pi R^2 \end{aligned}$$

Exercise 4

1. The full region between $r = a$ and $r = b$ may be viewed as many resistances dR connected in series, where each resistor is a thin spherical shell with radius r and thickness dr:

$$dR = \frac{\rho\, dr}{4\pi r^2}$$

The total resistance is found by summing up all these individual resistances, i.e., by integrating from $r = a$ to $r = b$:

$$\begin{aligned} R &= \int dR \\ &= \int_a^b \frac{\rho\, dr}{4\pi r^2} \\ &= \frac{\rho}{4\pi} |_a^b \left(-\frac{1}{r}\right) \\ &= \frac{\rho}{4\pi}\left(\frac{1}{a} - \frac{1}{b}\right) \end{aligned}$$

2. The given expression for the current I shows that we may here use Gauss' law for the electric field to determine I:

$$I = \frac{1}{\rho} \cdot \frac{Q}{\varepsilon_0}$$

We must assume that the charge entering into the inner conductor immediately distributes itself over the spherical surface at $r = a$ before is starts on its way through the material between $r = a$ and $r = b$.
The potential difference between the inner and outer conducting shell is easily determined, since we know the electric field $\boldsymbol{E}$:

$$\begin{aligned} \Delta V &= V_a - V_b \\ &= -\int_b^a E(r)\, dr \\ &= \frac{Q}{4\pi\varepsilon_0} |_b^a \frac{1}{r} \\ &= \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right) \end{aligned}$$

From these expressions, it follows that the resistance is

$$R = \frac{\Delta V}{I} = \frac{\rho}{4\pi}\left(\frac{1}{a} - \frac{1}{b}\right)$$