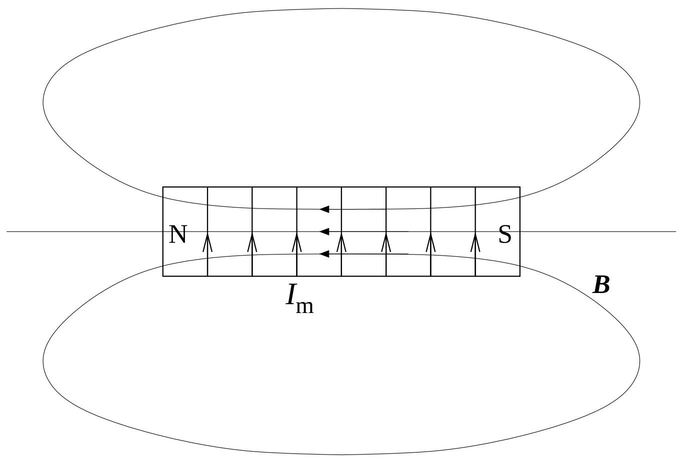
Institutt for fysikk, NTNU
TFY4155/FY1003 Elektrisitet og magnetisme
Vår 2005

Solution to øving 14

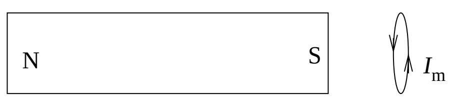
Guidance Thursday April 21 and Friday April 22

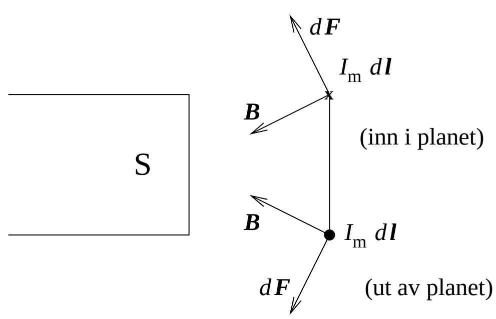
Exercise 1

a)



b) Let us imagine cylinder formed bar magnets. Such a magnet can then be viewed as a current carrying sylindrical "shell", alternatively closely spaced current carrying rings. We look at the magnetic force acting on such a ring in the magnetic field from the other bar magnet:





The force $d\boldsymbol{F}$ on a small element $d\boldsymbol{l}$ of the ring carrying a current I_m is, in the magnetic field $\boldsymbol{B}$, given by

$$d\boldsymbol{F} = I_m d\boldsymbol{l} \times \boldsymbol{B}$$

Two such forces are drawn in the figure above. We see that from symmetry arguments, the net force on such a ring must be towards the left. The same argument can be done for all the "rings" that together make up the whole bar magnet. Hence, we get *attraction* between the magnets.
Putting the two magnets S against S (or N against N) corresponds to a reversal of the current direction in the ring in the figure above. Then, we must also reverse the direction of all the contributions $d\boldsymbol{F}$ so that the total force on the magnet becomes to the right, i.e. *repulsion.*

c) An unmagnetized sphere made of steel contains a large number of ferromagnetic *domains*, where all atomic magnetic dipoles within a single domain point in the same direction, so that the magnetization $\boldsymbol{M}_d$ in the domain becomes nonzero. However, with no external magnetic field, $\boldsymbol{M}_d$ in different domains will point in various directions, so that the total magnetization in the sphere becomes zero. When the sphere enters the magnetic field of the bar magnet, the atomic dipole moments will be aligned along the external field, so that the whole sphere gets a magnetization $\boldsymbol{M}_k$ in the direction of the axis of the bar magnet. Now, we have essentially the same situation as in *b*) and can associate with the sphere a magnetization current in the surface, just as we did for the bar magnet. Hence, we get a net attraction betwee the bar magnet and the sphere.
It does not matter whether we put the sphere at the S- or the N-pole of the magnet. In both cases, the magnetic dipoles of the sphere will be aligned with the external field and give a net magnetization and corresponding magnetization current in the surface with direction so that the net force on the sphere becomes towards the magnet.

Exercise 2

a) We have used Ampere's law in the lectures to calculate the magnetic field inside a long solenoid:

$$B = \mu_0 n I_1 = \mu_0 \frac{N_1}{d} I_1$$

One winding of the solenoid wire encloses an area $A = \pi R^2$, and therefore a magnetic flux

$$\phi = BA = \mu_0 \frac{N_1}{d} I_1 \pi R^2$$

Then, N_1 windings must enclose a flux which is N_1 times bigger, because here, the magnetic field is constant everywhere inside the solenoid. Hence:

$$\phi_1 = N_1 \phi = \mu_0 \frac{N_1^2}{d} I_1 \pi R^2$$

One winding of solenoid 2 encloses exactly the same area, and therefore the same amount of flux ϕ, so that N_2 windings of solenoid 2 must enclose a total magnetic flux equal to

$$\phi_2 = N_2 \phi = \mu_0 \frac{N_1 N_2}{d} I_1 \pi R^2$$

b) The self inductance L becomes

$$L = \frac{\phi_1}{I_1} = \mu_0 \frac{N_1^2}{d} \pi R^2$$

c) Mutual inductance M becomes

$$M = \frac{\phi_2}{I_1} = \mu_0 \frac{N_1 N_2}{d} \pi R^2$$

d) Numerical values:

$$L = 4\pi \cdot 10^{-7} \cdot \frac{1200^2}{0.6} \cdot \pi \cdot 0.01^2 = 9.5 \cdot 10^{-4}$$

$$M = 4\pi \cdot 10^{-7} \cdot \frac{1200 \cdot 600}{0.6} \cdot \pi \cdot 0.01^2 = 4.7 \cdot 10^{-4}$$

In the SI system, inductance has its own unit, the henry (H). So, here the self inductance L is 0.95 mH and the mutual inductance M is 0.47 mH. Alternatively, we may use the unit T m^2/A, since magnetic flux must have the unit of magnetic field times area, i.e., T m^2.

Exercise 3

Here, we must find in which positions $\pm x_0$ we have turns that give zero x component to $\boldsymbol{B}_{\text{dipol}}$. The turns on the interval $(-x_0, x_0)$ will then be those that contribute with negative x component to $\boldsymbol{B}$.
From the given formula and the figure in the exercise, we have:

$$\begin{aligned} 3\,(\boldsymbol{m} \cdot \hat{r})\,\hat{r} - \boldsymbol{m} &= 3\left(m\,\hat{x} \cdot \left(\frac{x}{r}\hat{x} + \frac{y}{r}\hat{y}\right)\right)\left(\frac{x}{r}\hat{x} + \frac{y}{r}\hat{y}\right) - m\hat{x} \\ &= 3m\frac{x}{r}\frac{x}{r}\hat{x} + 3m\frac{x}{r}\frac{y}{r}\hat{y} - m\hat{x} \\ &= \left(3\frac{x^2}{r^2} - 1\right) m\hat{x} + 3\frac{xy}{r^2} m\hat{y} \end{aligned}$$

Here, we have expressed $\hat{r}$ in cartesian components, i.e., $\hat{r} = \cos\theta\,\hat{x} + \sin\theta\,\hat{y} = (x/r)\,\hat{x} + (y/r)\,\hat{y}$, where θ is the angle between $\boldsymbol{m}$ and $\hat{r}$.
Thus, zero x component when

$$3\frac{x^2}{r^2} - 1 = 0$$

and since $r^2 = x^2 + y^2$, we find

$$x_0 = y/\sqrt{2}$$

On the length $2x_0 = \sqrt{2}y = 0.707$ m, we have 707 turns.