Institutt for fysikk, NTNU
TFY4155/FY1003: Elektrisitet og magnetisme
Spring 2005

Summary, week 3 (January 18 and 19)

Electric field
[FGT 22.1; YF 21.4; TM 21.4; AF 21.5; LHL 19.4; DJG 2.1.3]

$$\boldsymbol{E} = \frac{\boldsymbol{F}}{q}$$

= force pr unit charge

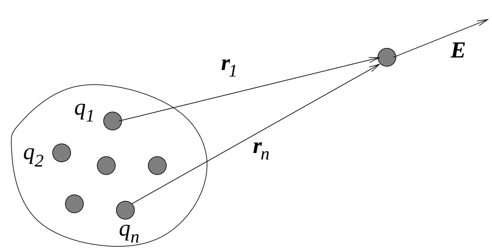
SI unit for electric field: $[E] = \mathrm{N/C}$

Electric field due to point charge
[FGT 22.1; YF 21.4; TM 21.4; AF 21.6; LHL 19.5; DJG 2.1.3]

$$\boldsymbol{E} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r}$$

The superposition principle for electric field:

$$\boldsymbol{E} = \sum_{j=1}^{n} \boldsymbol{E}_j = \frac{1}{4\pi\varepsilon_0}\sum_{j=1}^{n}\frac{q_j}{r_j^2}\hat{r}_j$$



Continuous charge distributions
[FGT 21.4, 22.3; YF 21.5; TM 22.1; AF eks. 21.6; LHL 19.5; DJG 2.1.4]

On a length scale which is large compared to the distance between single charges, one "sees" approximately a *continuous* charge distribution. (Analogy: Macroscopic objects have an approximately continuous mass distribution even though they are actually built up from "discrete masses" (i.e. atoms).)

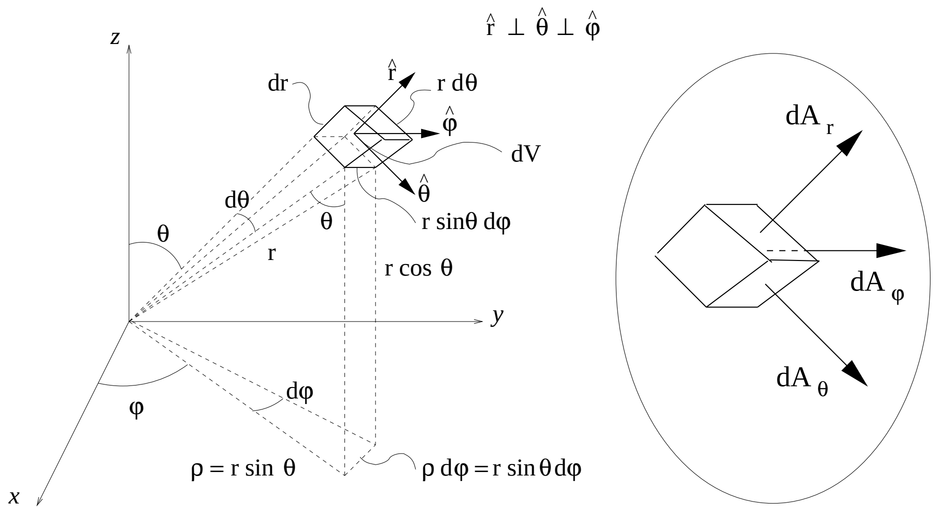
Sum over point charges is now replaced by an *integral* over a charge distribution:

$$\sum_i \Delta q_i \overset{\Delta q_i \to 0}{\to} \int dq$$

$3D$ (= 3 dimensions): volume charge

$$
\begin{aligned}
dq &= \rho\, dV \\
\rho &= \rho(x,y,z) = \text{ charge pr unit volume } = \text{ volume charge density} \\
[\rho] &= [q/V] = \mathrm{C/m^3} \\
\text{Volume element : } dV &= dx\, dy\, dz \;\; \text{(cartesian coordinates)} \\
&= r^2 \sin\theta\, d\theta\, d\phi\, dr \;\; \text{(spherical coordinates)} \\
&= \rho\, d\rho\, d\phi\, dz \;\; \text{(cylinder coordinates)}
\end{aligned}
$$

Volumelement dV i kulekoordinater:



dV = (dr) (r dθ) (r sin θ dφ)

$2D$: surface charge

$$
\begin{aligned}
dq &= \sigma\, dA \\
\sigma &= \sigma(x,y) = \text{ charge pr unit area } = \text{ surface charge density} \\
[\sigma] &= [q/A] = \mathrm{C/m^2} \\
\text{Surface element : } dA &= dx\, dy \;\; \text{(cartesian coordinates)} \\
&= r\, d\phi\, dr \;\; \text{(polar coordinates)}
\end{aligned}
$$

$1D$: line charge

$$
\begin{aligned}
dq &= \lambda\, dl \\
\lambda &= \lambda(x) = \text{ charge pr unit length } = \text{ line charge density} \\
[\lambda] &= [q/L] = \mathrm{C/m} \\
\text{Line element : } dl &= dx \;\; \text{(straight line)} \\
&= R\, d\phi \;\; \text{(for circle with radius } R\text{)}
\end{aligned}
$$

Electric field in a distance $\boldsymbol{r}$ from an infinitesimal charge dq:

$$d\boldsymbol{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\hat{r}$$

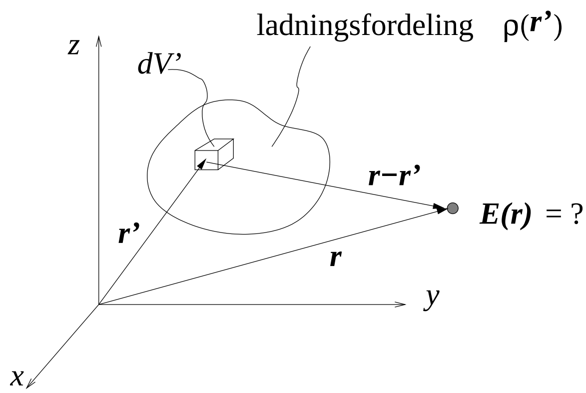
Electric field from continuous charge distribution:

$$\boldsymbol{E} = \frac{1}{4\pi\varepsilon_0}\int\frac{\hat{r}\,dq}{r^2} \overset{3D}{=} \frac{1}{4\pi\varepsilon_0}\int\frac{\hat{r}\rho\,dV}{r^2}$$

More precisely: The electric field $\boldsymbol{E}(\boldsymbol{r})$ in a position $\boldsymbol{r} = (x, y, z)$ due to a charge distribution described by the charge density $\rho(\boldsymbol{r}') = \rho(x', y', z')$ is given by

$$\boldsymbol{E}(\boldsymbol{r}) = \frac{1}{4\pi\varepsilon_0}\int\frac{(\boldsymbol{r}-\boldsymbol{r}')\rho(\boldsymbol{r}')\,dV'}{|\boldsymbol{r}-\boldsymbol{r}'|^3}$$

where $dV' = dx'\,dy'\,dz'$ (in cartesian coordinates) is a volume element in position $\boldsymbol{r}'$.
Note that $\boldsymbol{r}$ does not have the same meaning in the last two equations. In the first one, $\boldsymbol{r}$ denotes the vector from dq to the point where you are supposed to find $\boldsymbol{E}$. That means $\boldsymbol{r}$ is different for the different charge elements dq in the system we are looking at. In the second equation, $\boldsymbol{r}$ denotes the position where $\boldsymbol{E}$ is supposed to be determined, whereas $\boldsymbol{r}'$ is the position variable for the charge density ρ. We had a choice here concerning notation: We could have introduced a new vector $\boldsymbol{\mathcal{R}} \equiv \boldsymbol{r} - \boldsymbol{r}'$ and then write $\hat{\boldsymbol{\mathcal{R}}}/\mathcal{R}^2$, or, as we actually did above, rewrite the unit vector. Let's look at a figure:



We see that the unit vector in Coulombs law should point from the volume element dV' in position $\boldsymbol{r}'$ towards the position $\boldsymbol{r}$. Hence, we may write $(\boldsymbol{r}-\boldsymbol{r}')/|\boldsymbol{r}-\boldsymbol{r}'|^3$ in the expression for $\boldsymbol{E}(\boldsymbol{r})$.