Institutt for fysikk, NTNU
TFY4155/FY1003: Elektrisitet og magnetisme
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Summary, week 4 (January 25 and 26)

Electric field lines

[FGT 22.2; YF 21.6; TM 21.5; AF 21.6; LHL 19.6; DJG 2.2.1]

- provides a visual image of $\boldsymbol{E}$ in a given region of space
- $\boldsymbol{E}$ is everywhere tangential to the field lines
- the electric field strength (i.e. $|\boldsymbol{E}|$) is everywhere proportional to the density of field lines, i.e., number of field lines pr unit area

Consequences of this are, for instance:

- the field lines are directed radially *away from* positive (point-) charges and radially *towards* negative charges
- the same number of field lines start on a charge $+Q$ and end on a charge $-Q$

Electric dipole

[FGT 22.1; YF 21.7; TM 21.4; AF 21.11; LHL 19.10; DJG 2.2.1, 3.4.2]

If we have two charges q and $-q$ separated by some distance, we have an electric dipole. The (distance) vector $\boldsymbol{d}$ from the negative charge $-q$ to the positive charge q describes how the two charges are located with respect to each other.
The *dipole moment*, $\boldsymbol{p}$, of the dipole is then defined like this:

$$\boldsymbol{p} = q\boldsymbol{d}$$

Thus, the dipole moment is a *vector* pointing from the negative towards the positive charge, with magnitude equal to the product between the value of the charge q and the distance d.
Unit for electric dipole moment: $[p] = [qd] = \text{Cm}$.

Electric potential

[FGT 24.2; YF 23.2; TM 23.1; AF 21.9; LHL 19.9; DJG 2.3.1, 2.3.2, 2.4.1]

We have a *conservative* force $\boldsymbol{F}$ if the work $\int_A^B \boldsymbol{F}\cdot d\boldsymbol{l}$ is independent of the path followed between the starting position A and the end position B.
Examples of conservative forces are: The gravitational force between two masses. The electrostatic force between two charges.
Example of non-conservative force: Friction.

More generally, we have a *conservative vector field* $\boldsymbol{G}$ provided that the *line integral* $\int_A^B \boldsymbol{G} \cdot d\boldsymbol{l}$ is independent of the integration path between A and B.

For a conservative vector field $\boldsymbol{G}$, we always have:

$$\oint \boldsymbol{G} \cdot d\boldsymbol{l} = 0$$

where $\oint$ denotes integral around a *closed path* in space.

For a conservative force $\boldsymbol{F}$ we have a *potential energy* U so that the work done by $\boldsymbol{F}$ on "the system" (e.g. the charge being moved) by a displacement from A to B corresponds to the *change* in the potential energy of the system:

$$\Delta U = U_B - U_A = -\int_A^B \boldsymbol{F} \cdot d\boldsymbol{l}$$

[Check of the sign: A displacement of a mass m upwards, i.e., *against* the gravitational force $m\boldsymbol{g}$, results in an increase in the potential energy. At the same time, we have $\boldsymbol{F} \cdot d\boldsymbol{l} < 0$, which means that the sign is OK!]

Just as it was convenient to introduce the electric field $\boldsymbol{E} = \boldsymbol{F}/q =$ electric force pr unit charge, it is now convenient to introduce *electric potential* as *potential energy pr unit charge*:

$$V = U/q$$

Unit for electric potential: $[V] = [U/q] = \mathrm{J/C} \equiv \mathrm{V}$ (volt)

The relation between electric potential V and the electric field $\boldsymbol{E}$

[FGT 24.2; YF 23.2; TM 23.1; AF 21.10; LHL 19.9; DJG 2.3.1]

A charge q that is influenced by an electrostatic force $\boldsymbol{F}$ will have a difference in its potential energy

$$\Delta U = U_B - U_A = -\int_A^B \boldsymbol{F} \cdot d\boldsymbol{l}$$

between the two points A and B. Then the *difference in electric potential* ΔV between the points A and B must be

$$\Delta V = V_B - V_A = \frac{\Delta U}{q} = -\int_A^B \frac{\boldsymbol{F} \cdot d\boldsymbol{l}}{q} = -\int_A^B \boldsymbol{E} \cdot d\boldsymbol{l}$$

An alternative unit for electric field: $[E] = [V/l] = \mathrm{V/m}$

Note that while electric field is a *vector*, the electric potential is a *scalar*.

Note: Only *differences* in electric potential (and in potential energy) have physical meaning. We are free to choose where we want to have $V = 0$. A common choice is: $V(r \to \infty) = 0$. Then, for the potential in a point P:

$$V_P = -\int_\infty^P \boldsymbol{E} \cdot d\boldsymbol{l}$$

(We cannot always choose $V(r \to \infty) = 0$. We will see some examples of that later.)