Institutt for fysikk, NTNU
TFY4155/FY1003: Elektrisitet og magnetisme
Spring 2005

Summary, week 5 (February 1 and 2)

The energy unit electron volt

[FGT 24.2; YF 23.2; AF 21.9; LHL 19.9]

A *convenient* energy unit:
1 electron volt = 1 eV = $e \cdot 1\text{V} = 1.6 \cdot 10^{-19}\text{C}\cdot 1\ \text{J/C} = 1.6 \cdot 10^{-19}$ J
= change in potential energy for en elementary charge that is moved between two points with a potential difference of 1 volt
Why eV instead of J?

- differences between energy levels in atoms are of the order 1 eV
- even in particle physics and high energy physics, the particles usually have energies that are much smaller than 1 J, e.g. MeV (= 10^6 eV) or GeV (= 10^9 eV)
- thermal energies, $k_B T$, is at room temperature ($T = 300$ K) approximately 25 meV

In other words: The unit 1 J is "too large" in many branches of physics. However, the unit 1 eV (or meV, keV, MeV, GeV...) is more "suitable".

Electric potential from a point charge (the Coulomb potential)

[FGT 24.2; YF 23.2; TM 23.2; AF 21.11; LHL 19.9; DJG 2.3.4]:

$$V(r) = \frac{q}{4\pi\varepsilon_0 r}$$

Electric potential from continuous charge distributions:

$$V(r) = \frac{1}{4\pi\varepsilon_0} \int \frac{dq}{r}$$

with

$$dq = \begin{cases} \rho(x,y,z)\ dV & (3D) \\ \sigma(x,y)\ dA & (2D) \\ \lambda(x)\ dx & (1D) \end{cases}$$

Potential energy between point charges
[FGT 24.2; YF 23.1; TM 24.1; AF 21.9, 21.12; LHL 19.9; DJG 2.4]

- between two point charges in mutual distance r_{12}: $U_{12} = q_1 q_2 / 4\pi\varepsilon_0 r_{12}$
- between several point charges: $U = \sum_{i<j} U_{ij}$ (where the sum over i and j runs over all the point charges in the system, with the restriction that j is always larger than i, i.e., the sum runs over all pairs of point charges)

Energy conservation for charged particles in electric field
[FGT 24.1; YF 23.1; AF 21.12; LHL 19.9]

With *conservative* forces, the *total energy is conserved*:

$$\begin{aligned} T + U &= \text{constant} \\ T &= \frac{1}{2}mv^2 = \text{ kinetic energy} \\ U &= qV = \text{ potential energy} \end{aligned}$$

If a particle with charge q and mass m is accelerated in an electric field $\boldsymbol{E}$, i.e., through a potential difference $\Delta V = V_2 - V_1$:

$$\begin{aligned} \frac{1}{2}mv_1^2 + qV_1 &= \frac{1}{2}mv_2^2 + qV_2 \\ \Rightarrow v_2^2 &= v_1^2 + \frac{2q}{m}(V_1 - V_2) \\ \Rightarrow v_2 &= \sqrt{v_1^2 - \frac{2q\Delta V}{m}} = v_1\sqrt{1 - \frac{2q\Delta V}{mv_1^2}} \end{aligned}$$

Here, v_1 is the speed of the particle in a position where we have potential V_1 ("the initial point") and v_2 is the speed of the particle in a position where we have potential V_2 ("the final point"). If $q\Delta V < 0$, we obtain $v_2 > v_1$, i.e., a positive acceleration.
Hence:
Negative charges are accelerated in the direction of higher potential
Positive charges are accelerated in the direction of lower potential
.... while both types of charges of course are accelerated in the direction of lower potential energy.

The specific path followed by a particle with mass m and charge q in an electric field is determined by Newton's 2. law (= *the equation of motion*):

$$\boldsymbol{F} = m\boldsymbol{a} = m\frac{d\boldsymbol{v}}{dt} = q\boldsymbol{E}$$

Thus:

$$\boldsymbol{a} = \frac{q}{m}\boldsymbol{E}$$

So, a particle with positive charge is accelerated along $\boldsymbol{E}$.
A particle with negative charge is accelerated along $-\boldsymbol{E}$.

Equipotential surfaces
[FGT 24.3; YF 23.4; TM 23.5; AF 21.11; LHL 19.11; DJG 2.3.2]

Equipotential surfaces are surfaces in space with constant potential.

$$\Delta V = -\int \boldsymbol{E} \cdot d\boldsymbol{l} = 0$$

for an *arbitrary* integration path on an equipotential surface.
Hence:

$$\boldsymbol{E} \perp \quad \text{equipotentialsurface}$$

Calculation of $\boldsymbol{E}$ from V
[FGT 24.4; YF 23.5; TM 23.3; AF 21.10; LHL 19.9; DJG 2.3.1, 1.2.2]

$$\boldsymbol{E} = -\nabla V$$

The gradient operator ∇:

$$\begin{aligned} \nabla V &= \frac{\partial V}{\partial x}\hat{x} + \frac{\partial V}{\partial y}\hat{y} + \frac{\partial V}{\partial z}\hat{z} \quad \text{(cartesian coordinates)} \\ &= \frac{\partial V}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{\phi} \quad \text{(sphericalcoordinates)} \end{aligned}$$

(NB: You are not supposed to remember what the gradient operator looks like in spherical coordinates. If you will need it in e.g. an exam, it will be given.)
If we have spherical symmetry, i.e., $\boldsymbol{E}$ and V depend only on r, not the angles θ and ϕ:

$$\boldsymbol{E} = E(r)\hat{r} = -\frac{\partial V}{\partial r}\hat{r}$$

Next week, we will learn that:
The vector ∇V points in the direction where V increases the most. Since $\boldsymbol{E} = -\nabla V$, this means that the electric field points in the direction where V decreases the most.
We will also take a look at an example where we first calculate V and then $\boldsymbol{E}$.
After that, we will start with electric flux and Gauss' law for the electric field.