Institutt for fysikk, NTNU
TFY4155/FY1003: Elektrisitet og magnetisme
Spring 2005

Summary, week 8 (February 22 and 23)

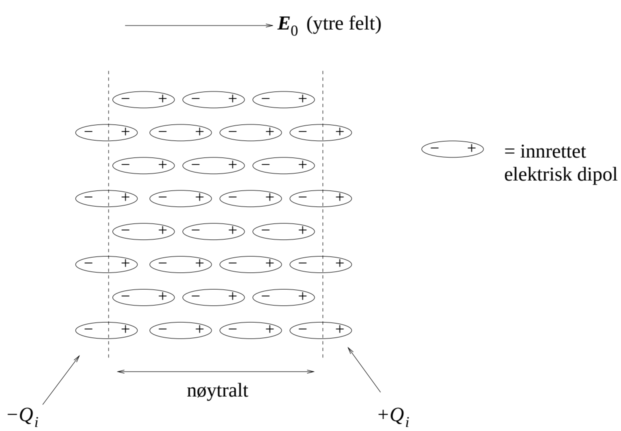
Electric polarization. Dielectrics.

[FGT 25.5, 25.6; YF 24.4, 24.5; TM 24.5, 24.6; AF 25.6, 25.7; LHL 20.5; DJG 4.1]

Insulator: No mobile (free) charges (but *bound* charges)
Dielectric: Polarizable insulator. (All insulators are polarizable.)

If a dielectric is placed in an external electric field $\boldsymbol{E}_0$, we obtain alignment of (molecular) electric dipoles along $\boldsymbol{E}_0$, cf øving 6, exercise 2. (Alternatively: Polarization internally in atoms and nonpolar molecules that, at the outset, have zero electric dipole moment.)

Net (macroscopic) effect of the external field:
Displacement of bound charge.



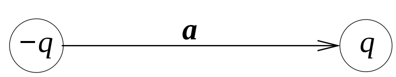
$\pm Q_i$ = net induced charge on the surface of the insulator.

Polarization = dipole moment pr unit volume:

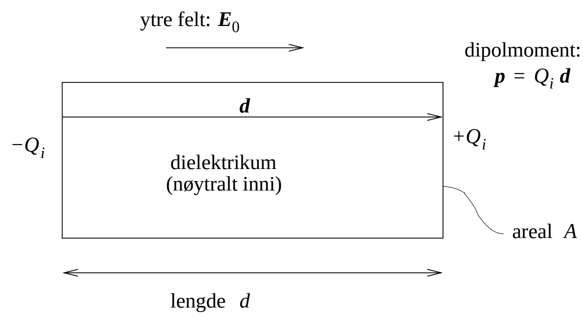
$$\boldsymbol{P} = \frac{\boldsymbol{p}}{V}$$

Electric dipole moment (repetition!):

$$\boldsymbol{p} = q\boldsymbol{a}$$



Dielectric in external field $\boldsymbol{E}_0$:



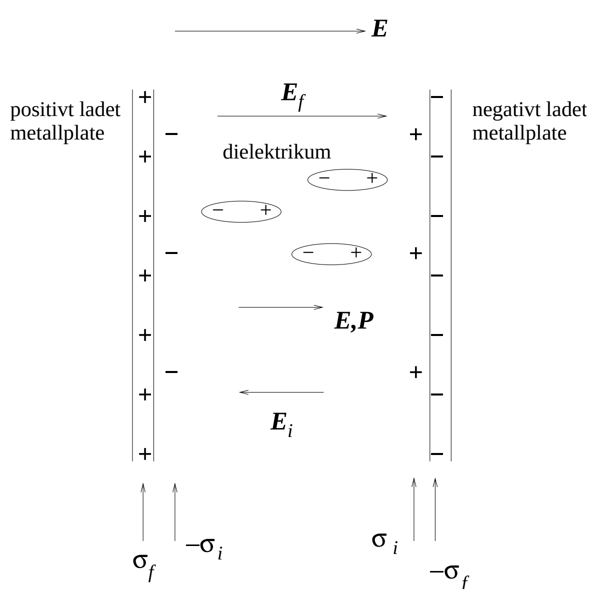
Volume: $V = Ad$
Density of induced surface charge: $\sigma_i = Q_i/A$

Hence:
Total dipole moment: $p = |\boldsymbol{p}| = Q_i d = \sigma_i A d = \sigma_i V$
Polarization: $P = |\boldsymbol{P}| = p/V = \sigma_i$

In general: $\boldsymbol{P} \cdot \hat{n} = P_\perp = \sigma_i$
($\hat{n}$ = surface normal, $P_\perp$ = component of $\boldsymbol{P}$ perpendicular to the surface)

Electric displacement.
[FGT 25.6; YF 24.6; TM 24.6; AF 25.8; LHL 20.5; DJG 4.3]

We use the idealized system which we have used before:
Oppositely charged metal plates (infinitely large), now with dielectric in between:



Free charge pr unit area on the metal plates: σ_f

... which generate electric field between the plates: $E_f = \sigma_f/\varepsilon_0$ (zero field outside the plates)
Induced charge pr unit area on the surface of the dielectric: σ_i
... which generate electric field between the plates: $E_i = \sigma_i/\varepsilon_0$
Total field between the plates: $\boldsymbol{E} = \boldsymbol{E}_f + \boldsymbol{E}_i \quad \Rightarrow \quad E = |\boldsymbol{E}| = E_f - E_i = (\sigma_f - \sigma_i)/\varepsilon_0$
Net charge on the surfaces: $\pm\sigma = \pm(\sigma_f - \sigma_i)$
... which generate total field between the plates: $E = \sigma/\varepsilon_0 = (\sigma_f - \sigma_i)/\varepsilon_0$, OK!

We have: $\sigma_i = P =$ polarization in the dielectric (= dipole moment pr unit volume)

Hence:

$$\sigma_f = \sigma + \sigma_i = \varepsilon_0 E + P$$

So we see that the density of *free* charge σ_f is determined by the combination $\varepsilon_0 E + P$. In many situations, e.g. in experiments, it is precisely the free charge that we are able to control. With dielectrics present, it is therefore often *convenient* to "refer to" the vector field

$$\boldsymbol{D} \equiv \varepsilon_0 \boldsymbol{E} + \boldsymbol{P}$$

$\boldsymbol{D}$ is called *electric displacement*.

Here:

$$D = |\boldsymbol{D}| = \sigma_f$$

In general (as we found for $\boldsymbol{P}$):

$$\sigma_f = \boldsymbol{D} \cdot \hat{n} = D_\perp$$

where $D_\perp$ is the normal component of the electric displacement.

Gauss' lov for $\boldsymbol{D}$:

$$\oint_S \boldsymbol{D} \cdot d\boldsymbol{A} = Q_f$$

where Q_f is net *free* chrage inside the closed surface S. (Net total charge inside S is $Q_{\text{in}} = Q_f - Q_i$, with $-Q_i =$ net *bound* charge, associated with the polarization $\boldsymbol{P}$, inside S.)

Electric susceptibility and permittivity.
[FGT 25.5; YF 24.4; TM 24.5, 24.6; AF 25.9; LHL 20.5; DJG 4.4]

Linear response: $\boldsymbol{P}$ proportional to $\boldsymbol{E}$, i.e., we may write

$$\boldsymbol{P} = \chi_e \varepsilon_0 \boldsymbol{E}$$

where we have introduced $\chi_e =$ electric susceptibility.
NB: Such a linear relation between $\boldsymbol{P}$ and $\boldsymbol{E}$ is not always valid, but in this course, we will assume that it is valid. Also note that $\boldsymbol{E}$ is the *total* field, and not only the external field.

Hence:

$$\begin{aligned} \boldsymbol{D} &= \varepsilon_0 \boldsymbol{E} + \boldsymbol{P} \\ &= (1 + \chi_e)\varepsilon_0 \boldsymbol{E} \\ &= \varepsilon_r \varepsilon_0 \boldsymbol{E} \\ &= \varepsilon \boldsymbol{E} \end{aligned}$$

Here, we have introduced
$\varepsilon_r = 1 + \chi_e$ = relative permittivity ("the dielectric constant")
$\varepsilon = \varepsilon_r \varepsilon_0$ = the permittivity of the medium

Units:
$[\chi_e] = [\varepsilon_r] = 1$ (dimensionless)
$[\varepsilon] = [\varepsilon_0] = \text{C}^2/\text{Nm}^2$

Point charge q in dielectric with permittivity ε:

Electric field: $\boldsymbol{E}(r) = (q/4\pi\varepsilon r^2)\hat{r}$
Electric potential: $V(r) = q/4\pi\varepsilon r$
I.e.: As for point charge in vacuum, but with $\varepsilon_0 \to \varepsilon > \varepsilon_0$; the medium is polarized and *screens* the point charge so that E and V are reduced by the factor $1/\varepsilon_r$.

Capacitor and capacitance.
[FGT 25.1, 25.5; YF 24.1, 24.2; TM 24.4; AF 25.10; LHL 20.1; DJG 2.5.4]

Capacitor = two separated electric conductors with charge $\pm Q$ (Or sometimes: One electric conductor with charge Q, with the other conductor moved to infinity.)

Coulomb's law $\Rightarrow$ electric field around the conductors is proportional with Q

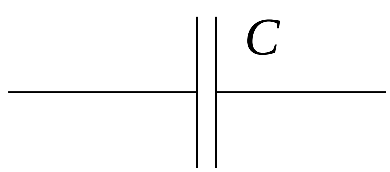
It then follows that the *potential difference* ΔV between the two conductors is also proportional with Q:

$$C = \frac{Q}{\Delta V}$$

C = the *capacitance* of the conductor

Unit for capacitance: $[C] = [Q/\Delta V] = \text{C/V} \equiv \text{F}$ (farad)

Symbol in electric circuits:



C is a *geometric factor*, dependent on the shape and size of the two conductors, and their mutual distance. C also depends on what kind of medium we have in the region between the two conductors.
Capacitance is, by definition, a *positive* quantity.
Calculation of C for a given system is done by calculating the potential difference between the two conductors, $\Delta V = V_+ - V_-$, for a given charge $\pm Q$.

Parallel plate capacitor, filled with air (vacuum), with surface area A, and plate distance d:

$$C = \varepsilon_0 \frac{A}{d}$$

Parallel plate capacitor, filled with dielectric with relative permittivity ε_r, surface area A, and plate distance d:

$$C = \varepsilon_0 \varepsilon_r \frac{A}{d}$$

If the region between the two conductors of the capacitor is first air and then partly or fully filled with a dielectric, the capacitance will always increase. (The same happens if parts of the volume are filled with metal.)

Energy associated with electric field
[FGT 25.3; YF 24.3; TM 24.3; AF 25.11; LHL 20.4; DJG 2.4.3]

Density of potential energy, i.e., potential energy pr unit volume, is with electric field E equal to

$$u = \frac{1}{2}\varepsilon_0 E^2$$

We also found that the potential energy could be associated with the electric charge: If a system has electric potential $v(q)$ when the charge is q, we must perform a work $dW = v(q)\, dq$ in order to increase the charge from q to $q + dq$. Hence, the total work needed, and therefore also the total stored potential energy in the system, is

$$W = U = \int_0^Q v(q)\, dq$$

i.e., in order to charge the system from zero charge to a final charge Q.

Alternatively, we may calculate the stored potential energy by integrating the energy density u over the whole volume V:

$$U = \int_V u\, dV = \int_V \frac{1}{2}\varepsilon_0 E^2\, dV$$

(Note that V is here *volume* and not potential. Further: Don't be confused by the notation used above: I used $v(q)$ to denote the potential difference between the two capacitor plates when they had charge q and $-q$, i.e., at some arbitrary stage during the charging. The reason for the small v was I wanted to reserve V for the potential difference between the plates after they had been fully charged, i.e., with charge Q and $-Q$. I try to minimize mixup in the notation, but V is an exception, being used both for potential and for volume.)