NORGES TEKNISK-
NATURVITENSKAPELIGE UNIVERSITET
INSTITUTT FOR FYSIKK

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EXAM
FY1003 ELEKTRISITET OG MAGNETISME I
TFY4155 ELEKTROMAGNETISME
Friday June 8 2007, 0900 - 1300
English

Remedies: C

- K. Rottmann: Matematisk formelsamling
- O. Øgrim and B. E. Lian: Størrelser og enheter i fysikk og teknikk, or B. E. Lian and C. Angell: Fysiske størrelser og enheter.
- Approved calculator, with empty memory, according to list composed by NTNU (HP30S or similar.)

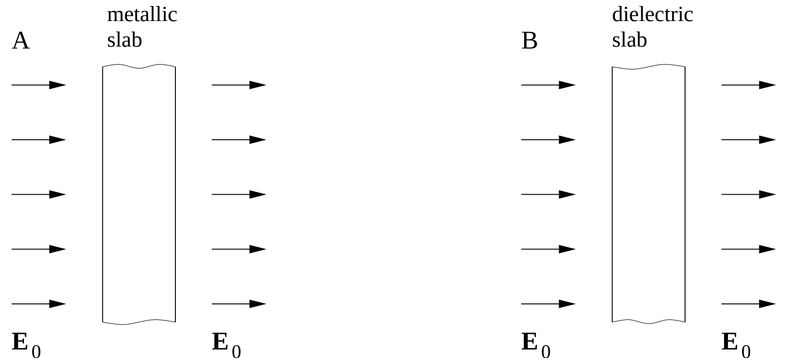
Pages 2 - 5: Questions 1 - 4.
Appendix 1 - 3: Formulas.

The exam consists of 10 partial questions (1a, 1b, 1c, 2a, 2b, 2c, 3a, 3b, 4a, 4b). Each of these 10 partial questions will be given equal weight during the grading. Vectors are given with **bold** letters. Unit vectors are given with a hat above the symbol. If nothing else is stated, you may assume that the surrounding medium is air (vacuum), with permittivity $\varepsilon_0 = 8.85 \cdot 10^{-12}$ F/m and permeability $\mu_0 = 4\pi \cdot 10^{-7}$ H/m. In questions where numerical values are provided for all necessary parameters, numerical answers are required.

The grades will be available no later than June 29.

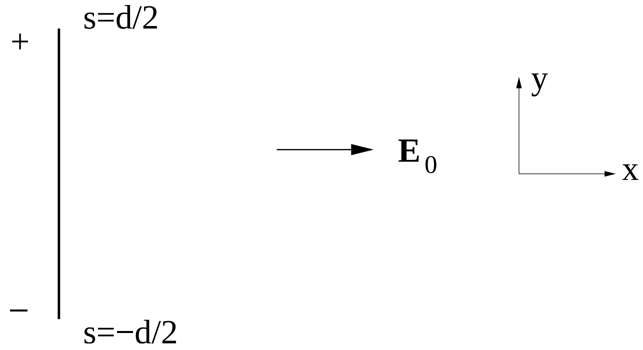
QUESTION 1

a) Explain briefly what happens, and why, when a metallic slab is placed across in a uniform external electric field $\boldsymbol{E}_0$ (figure A below). Correspondingly, explain what happens, and why, when a dielectric slab is placed across in a uniform external electric field $\boldsymbol{E}_0$ (figure B below). For simplicity, assume that the slabs have a large area perpendicular to the direction of the external field. ("what happens" here refers to electric charges and electric field in the two slabs.)



b) A thin rod has length d and line charge (charge pr unit length) $\lambda(s) = 2\lambda_0 s/d$. Here, λ_0 is a constant, and s denotes the position along the rod, from $s = -d/2$ in one end to $s = d/2$ in the other. In other words, this is an electric dipole, with zero net charge. Find the electric dipole moment p of the rod. (Hint: Charges dq and $-dq$ separated by a distance $2s$ contribute with dipole moment $dp = 2s\,dq$.)

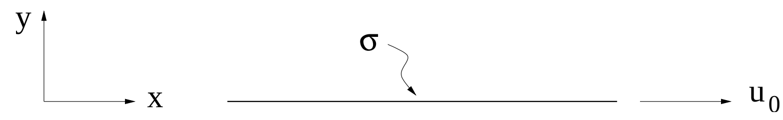
c) The rod in question *b*) is placed in a uniform external electric field $\boldsymbol{E}_0 = E_0\,\hat{x}$ such that the dipole moment $\boldsymbol{p}$ points in the positive y direction. Find the torque $\boldsymbol{\tau} = \int (\boldsymbol{r} \times d\boldsymbol{F})$ (both absolute value and direction) that acts on the rod.



Show with a figure which orientations of the rod in the external field that represent, respectively, (1) maximum potential energy; (2) minimum potential energy; (3) maximum torque.

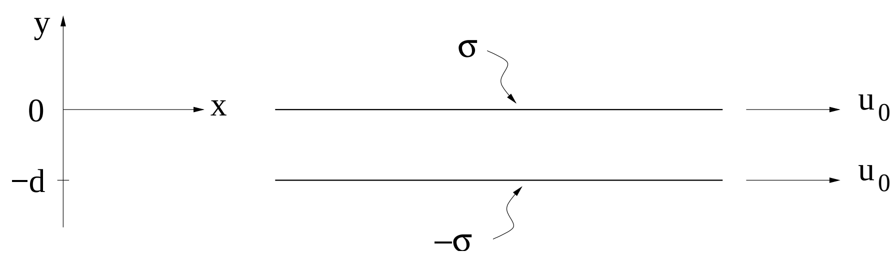
QUESTION 2

a) An infinitely large plane has uniform (and positive) charge σ pr unit area and is located in the xz plane. The plane with the charges moves with constant velocity $u_0\,\hat{x}$, i.e., in the positive x direction.



What is the direction of the resulting electric field $\boldsymbol{E}$ and magnetic field $\boldsymbol{B}$, both above ($y > 0$) and below ($y < 0$) the charged plane? Use Gauss' law and Ampere's law to show that the fields are uniform on both sides of the plane, with field strength E_0 and B_0, respectively. Find expressions for E_0 and B_0 (i.e., in terms of given parameters and possibly some natural constants). Draw figures that clearly illustrate how you have used Gauss' law and Ampere's law.

b) Two parallel — and infinitely large — planes have uniform charge σ and $-\sigma$, respectively, pr unit area, and both planes have their surface normal in the y direction. The positively charged plane is located in $y = 0$ (as in question *a*)) whereas the negatively charged plane is located in $y = -d$. Both planes move with constant velocity $u_0\,\hat{x}$:



Find the resulting fields $\boldsymbol{E}$ and $\boldsymbol{B}$ in the three regions $y > 0$, $0 > y > -d$, and $y < -d$. The presence of these fields means that the two charged planes influence each other with an electric force f_E and a magnetic force f_B pr unit area. Determine for each of these forces whether it is attractive or repulsive. Find the ratio $\kappa = |f_B/f_E|$ between magnetic and electric force when the speed of the planes is $u_0 = 3 \cdot 10^6$ m/s. (Express your answers in this question in terms of E_0 and B_0 if you did not manage to determine them in question *a*).)

c) A particle with mass m and charge $q > 0$ starts at the origin with zero initial velocity at time $t = 0$. The particle finds itself in "crossed" uniform electric and magnetic fields, $\boldsymbol{E} = E\,\hat{y}$ and $\boldsymbol{B} = B\,\hat{z}$, respectively. Use Newton's second law to show that the motion of the particle is described by the equations

$$\begin{aligned} \frac{d^2 v_x}{dt^2} + \omega^2 v_x &= \frac{\omega^2 E}{B}, \\ \frac{d^2 v_y}{dt^2} + \omega^2 v_y &= 0, \end{aligned}$$

and find thereby the "cyclotron frequency" ω. The path of the particle becomes

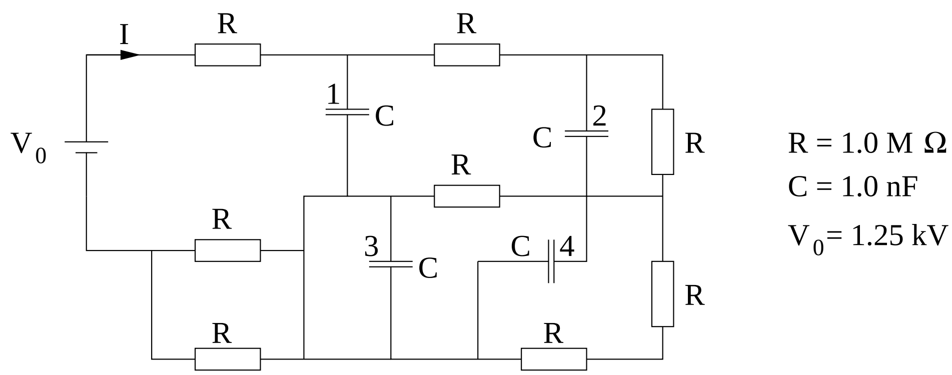
$$\begin{aligned} x(t) &= r_0\,(\omega t - \sin\omega t)\,, \\ y(t) &= r_0\,(1 - \cos\omega t)\,. \end{aligned}$$

Show that this path is a solution of the equations of motion with the given initial conditions $x(0) = y(0) = 0$ and $v_x(0) = v_y(0) = 0$. What is r_0?

Given information: $\boldsymbol{F} = q\,(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$ (The Lorentz force)

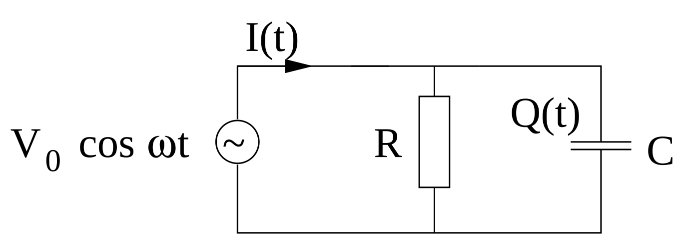
QUESTION 3

a) In the circuit below, the DC voltage source V_0 has been connected for so long that currents in the circuit and charges on the capacitors no longer change with time.



Find the current I, and also the charges Q_1, Q_2, Q_3, and Q_4 on the capacitors marked 1, 2, 3, and 4.

b) In the circuit below, an AC voltage source $V(t) = V_0 \cos\omega t$ is connected to a resistance R and a capacitor with capacitance C, with R and C in parallel.



We assume that the voltage source has been connected for such a long time that the current $I(t)$ oscillates with the same angular frequency as the voltage source. The same is true for the charge $Q(t)$ and the currents $I_R(t)$ through the resistor and $I_C(t)$ in and out of the capacitor.

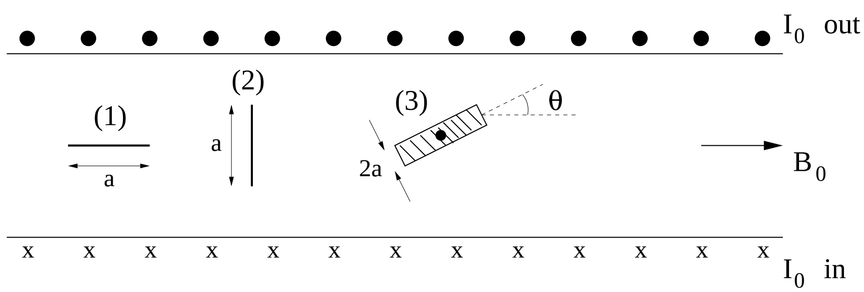
Use the principles of charge and energy conservation (i.e., Kirchhoff's current and voltage rule) to find $Q(t)$, $I_R(t)$, and $I_C(t)$. Write the total current "delivered" by the voltage source on the form

$$I(t) = I_0 \cos(\omega t - \alpha)$$

and determine the phase difference α between current and voltage, and the "generalized resistance" (the impedance) $Z(\omega) = V_0/I_0(\omega)$. Check that your answer for $Z(\omega)$ is reasonable when the angular frequency ω becomes very small ($\omega \ll 1/RC$).

Given information: $\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$

QUESTION 4



Inside an appoximately infinitely long and tightly wound solenoid with n turns pr unit length, we have put (1) a quadratic current loop with edges a and surface normal perpendicular to the symmetry axis of the long solenoid; (2) a quadratic current loop with edges a and surface normal parallel with the symmetry axis of the long solenoid; (3) a small cylindrical solenoid with radius a, N turns, and cylinder axis making an angle θ with the symmetry axis of the long solenoid.

a) Find the mutual inductance M_i ($i = 1, 2, 3$) between the long solenoid and each of the three conductors placed inside.

b) A current I_0 runs in the wire of the long solenoid (see figure). The small solenoid (3) rotates with angular frequency ω around an axis pointing out of the paper plane, and passing through the center of the solenoid (3). Find an expression for the induced electromotive voltage $\mathcal{E}_3(t)$ in the solenoid (3). Find the amplitude of $\mathcal{E}_3(t)$ when $n = 10^4$ m^{-1}, $N = 50$, $a = 1.0$ cm, $I_0 = 6.0$ A, and $\omega = 10^3$ s^{-1}.

Given information: $B_0 = \mu_0 n I_0$

Formulas

$\int d\boldsymbol{A}$ denotes surface integral and $\int d\boldsymbol{l}$ denotes line integral. $\oint$ denotes integral over closed surface or around closed curve. The validity of the formulas and the meaning of the various symbols are assumed to be known.

Electrostatics

- Coulomb's law:

$$\boldsymbol{F} = \frac{qq'}{4\pi\varepsilon_0 r^2}\hat{r}$$

- Electric field and potential:

$$\boldsymbol{E} = -\nabla V$$

$$\Delta V = V_B - V_A = -\int_A^B \boldsymbol{E} \cdot d\boldsymbol{l}$$

- Electric potential from point charge:

$$V = \frac{q}{4\pi\varepsilon_0 r}$$

- Electric flux:

$$\phi_E = \int \boldsymbol{E} \cdot d\boldsymbol{A}$$

- Gauss' law for electric field:

$$\varepsilon_0 \oint \boldsymbol{E} \cdot d\boldsymbol{A} = q$$

$$\oint \boldsymbol{D} \cdot d\boldsymbol{A} = q_{\text{free}}$$

- Electrostatic field is conservative:

$$\oint \boldsymbol{E} \cdot d\boldsymbol{l} = 0$$

- Electric displacement:

$$\boldsymbol{D} = \varepsilon_0 \boldsymbol{E} + \boldsymbol{P} = \varepsilon_r \varepsilon_0 \boldsymbol{E} = \varepsilon \boldsymbol{E}$$

- Electric dipole moment:

$$\boldsymbol{p} = q\boldsymbol{d}$$

- Electric polarization = electric dipole moment pr unit volume:

$$\boldsymbol{P} = \frac{\Delta \boldsymbol{p}}{\Delta V}$$

- Capacitance:

$$C = \frac{q}{V}$$

- Energy density in electric field:

$$u_E = \frac{1}{2}\varepsilon_0 E^2$$

Magnetostatics

- Magnetic flux:

$$\phi_m = \int \boldsymbol{B} \cdot d\boldsymbol{A}$$

- Gauss' law for the magnetic field:

$$\oint \boldsymbol{B} \cdot d\boldsymbol{A} = 0$$

- Ampère's law:

$$\oint \boldsymbol{B} \cdot d\boldsymbol{l} = \mu_0 I$$

$$\oint \boldsymbol{H} \cdot d\boldsymbol{l} = I_{\text{free}}$$

- Magnetic field from current carrying conductor (Biot–Savart law):

$$\boldsymbol{B} = \frac{\mu_0}{4\pi} I \int \frac{d\boldsymbol{l} \times \hat{r}}{r^2}$$

- The $\boldsymbol{H}$-field:

$$\boldsymbol{H} = \frac{1}{\mu_0}\boldsymbol{B} - \boldsymbol{M} = \frac{1}{\mu_r \mu_0}\boldsymbol{B} = \frac{1}{\mu}\boldsymbol{B}$$

- Magnetic dipole moment:

$$\boldsymbol{m} = I\boldsymbol{A}$$

- Magnetization = magnetic dipole moment pr unit volume:

$$\boldsymbol{M} = \frac{\Delta \boldsymbol{m}}{\Delta V}$$

- Magnetic force on straight current carrying conductor:

$$\boldsymbol{F} = I\boldsymbol{L} \times \boldsymbol{B}$$

- Energy density in magnetic field:

$$u_B = \frac{1}{2\mu_0} B^2$$

Electrodynamics and electromagnetic induction

- Faraday–Henry's law:

$$\mathcal{E} = \oint \boldsymbol{E} \cdot d\boldsymbol{l} = -\frac{d\phi_m}{dt}$$

- Ampère–Maxwell's law:

$$\oint \boldsymbol{B} \cdot d\boldsymbol{l} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\phi_E}{dt}$$

- Selfinductance:

$$L = \frac{\phi_m}{I}$$

- Mutual inductance:

$$M_{12} = \frac{\phi_1}{I_2} \;\; , \;\; M_{21} = \frac{\phi_2}{I_1} \;\; , \;\; M_{12} = M_{21} = M$$

- Energy density in electromagnetic field:

$$u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0} B^2$$