Institutt for fysikk, NTNU
FY1003 Elektrisitet og magnetisme I
TFY4155 Elektromagnetisme
Spring 2007

Midterm exam Wednesday March 7 2007, 1300 – 1500.

Fill in the table on page 11. Mark your answers clearly.
Remember to fill in your student number, both on page 3 and page 11.
Deliver both the table with your answers and the exercise text.

Allowed remedies: C

- K. Rottmann: Matematisk formelsamling. (Or similar.)
- O. Øgrim and B. E. Lian: Størrelser og enheter i fysikk og teknikk or B. E. Lian and C. Angell: Fysiske størrelser og enheter.
- Approved calculator, with empty memory, according to list provided by NTNU. (HP30S or similar.)
- Formulas, Electrostatics is included on page 2.

Given information:

- The exam consists of 25 questions. Each question has one correct and three incorrect answers.
- Choose *one* alternative for *each* question. If you choose *more than one* alternative or *no* alternative, the answer will be considered wrong.
- Unless stated otherwise, we assume that the system is in electrostatic equilibrium.
- Unless stated otherwise, "potential" means "electrostatic potential", and analogously for "potential energy".
- Unless stated otherwise, zero potential and potential energy is chosen at infinity.
- Metal is synonymous with electric conductor. Insulator is synonymous with dielectric.
- Some fundamental constants: $\varepsilon_0 = 8.85{\cdot}10^{-12}$ C^2/Nm^2, $1/4\pi\varepsilon_0 = 9{\cdot}10^9$ Nm^2/C^2, $e = 1.6{\cdot}10^{-19}$ C, $m_e = 9.11 \cdot 10^{-31}$ kg, $m_p = 1.67 \cdot 10^{-27}$ kg, $g = 9.8$ m/s^2, $c = 3 \cdot 10^8$ m/s.
- Symbols are given in italics (e.g. V for potential) while units are given in non-italics (e.g. V for volt).
- SI-prefixes: M (mega) = 10^6, k (kilo) = 10^3, c (centi) = 10^{-2}, m (milli) = 10^{-3}, μ (micro) = 10^{-6}, n (nano) = 10^{-9}, p (pico) = 10^{-12}.
- Circumference of circle: $2\pi r$. Area of spherical surface: $4\pi r^2$. Volume of sphere: $4\pi r^3/3$.
- Gradient in cartesian coordinates: $\nabla f = (\partial f/\partial x)\,\hat{x} + (\partial f/\partial y)\,\hat{y} + (\partial f/\partial z)\,\hat{z}$
- Gradient of spherically symmetric function $f(r)$: $\nabla f = (\partial f/\partial r)\,\hat{r}$
- Some integrals: $\int x^n\,dx = x^{n+1}/(n+1) + C$, $\int dx/x = \ln|x| + C$, $\int_0^{2\pi} \cos^2 x\,dx = \pi$

Formulas, Electrostatics

$\int d\boldsymbol{A}$ denotes surface integral and $\int d\boldsymbol{l}$ denotes line integral. $\oint$ denotes integral over closed surface or around closed curve. **Boldface** symbols denote vectors. Symbols with a "hat" above denote unit vectors. The validity of the formulas and the meaning of the various symbols are assumed to be known.

- Coulomb's law:
$$\boldsymbol{F} = \frac{qq'}{4\pi\varepsilon_0 r^2}\hat{r}$$
- Electric field and potential:
$$\boldsymbol{E} = -\nabla V$$
$$\Delta V = V_B - V_A = -\int_A^B \boldsymbol{E}\cdot d\boldsymbol{l}$$
- Electric potential from point charge:
$$V = \frac{q}{4\pi\varepsilon_0 r}$$
- Electric flux:
$$\phi_E = \int \boldsymbol{E}\cdot d\boldsymbol{A}$$
- Electrostatic force is conservative:
$$\oint \boldsymbol{E}\cdot d\boldsymbol{l} = 0$$
- Gauss' law for electric field and electric displacement:
$$\varepsilon_0 \oint \boldsymbol{E}\cdot d\boldsymbol{A} = q$$
$$\oint \boldsymbol{D}\cdot d\boldsymbol{A} = q_{\text{free}}$$
- Electric displacement:
$$\boldsymbol{D} \equiv \varepsilon_0\boldsymbol{E} + \boldsymbol{P} = \varepsilon_r\varepsilon_0\boldsymbol{E} = \varepsilon\boldsymbol{E}$$
- Electric dipole moment; in general, for region Ω with distribution of charge:
$$\boldsymbol{p} = \int_\Omega \boldsymbol{r}\, dq$$
- Electric dipole moment; for point charges $\pm q$ separated by distance $\boldsymbol{d}$:
$$\boldsymbol{p} = q\boldsymbol{d}$$
- Electric polarization = electric dipole moment pr unit volume:
$$\boldsymbol{P} = \frac{\Delta\boldsymbol{p}}{\Delta V}$$
  Linear response:
$$\boldsymbol{P} = \varepsilon_0\chi_e\boldsymbol{E}$$
- Capacitance:
$$C = \frac{q}{V}$$
- Energy density (energy pr unit volume) in electric field:
$$u_E = \frac{1}{2}\varepsilon_0 E^2$$

Course code: Student number:

Questions – Version A

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1) Which statement about electric potential is wrong?

A If the electric field in a region is uniform, the potential in this region is constant.
B The superposition principle is valid for electric potential.
C Electric potential may be measured in the unit C/F.
D A piece of metal with an empty cavity inside has the same potential on the outer surface as inside the cav

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2) Which statement about electric charge is correct?

A Net charge on a metal sphere always lies on the surface.
B Net charge on a plastic sphere always lies on the surface.
C Net charge on a metal sphere never lies on the surface.
D Net charge on a plastic sphere never lies on the surface.

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3) Which statement about a charged conductor is wrong?

A Inside the conductor, the electric field is zero.
B The conductor is an equipotential.
C On the surface of the conductor, the electric field is zero.
D Inside the conductor, the net charge is zero.

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4) Which statement is correct? The capacitance of a parallel plate capacitor ...

A ... is inversely proportional to the potential difference between the plates.
B ... is proportional to the charge on the plates.
C ... becomes smaller if we increase the distance between the plates.
D ... is independent of the choice of material between the plates.

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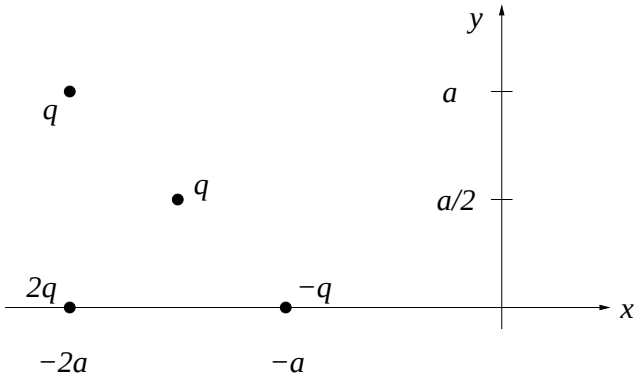
5) What is the force on the charge q located in position $(x, y) = (-3a/2, a/2)$?

A $(\hat{x} + \hat{y})\sqrt{2}q^2/\pi\varepsilon_0 a^2$
B $(\hat{x} - \hat{y})\sqrt{2}q^2/\pi\varepsilon_0 a^2$
C $\hat{y}\sqrt{2}q^2/\pi\varepsilon_0 a^2$
D $\hat{x}\sqrt{2}q^2/\pi\varepsilon_0 a^2$



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6) What is the total potential energy of the four point charges in question 5?

A $q^2/4\pi\varepsilon_0 a^2$
B $3q^2/4\sqrt{2}\pi\varepsilon_0 a$
C $3q/\sqrt{2}\pi\varepsilon_0 a$
D 0

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7) Three of the charges in question 5 are held fixed while the fourth charge, the upper left one, with charge q and mass m, is released with zero initial velocity from the position $(-2a, a)$. What is the speed v of this charge when it has come very far away from the other three?

A $v = \left[\left(\frac{\sqrt{2}}{4} + 1\right) q^2/m\pi\varepsilon_0 a\right]^{1/2}$
B $v = 0$
C $v = \left[\left(\frac{\sqrt{2}}{4} - 1\right) q^2/m\pi\varepsilon_0 a\right]^{1/2}$
D $v = \left[\left(1 - \frac{\sqrt{2}}{4}\right) q^2/m\pi\varepsilon_0 a\right]^{1/2}$

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8) Two small metal spheres have charges 6.0 μC and -5.0 μC, respectively. The distance between the spheres is 60 cm. The mutual force between the two spheres is then

A 0.50 N
B 0.75 N
C 1.00 N
D 1.25 N

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9) At a large distance $\boldsymbol{r} = L\,\hat{x}$ from a small (i.e.: with an extent much smaller than L) electric dipole with dipole moment $\boldsymbol{p} = p_0\,\hat{y}$ the electric field is $-E_0\,\hat{y}$. The field at a distance $3L\,\hat{x}$ is then approximately

A $-0.037E_0\,\hat{y}$
B $-0.11E_0\,\hat{y}$
C $-0.33E_0\,\hat{y}$
D $-E_0\,\hat{y}$

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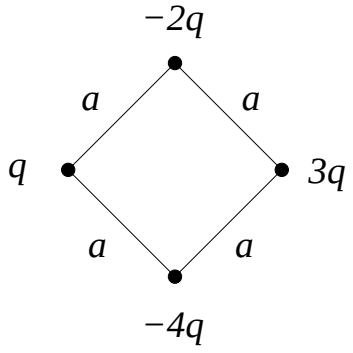
10) What is the electric field strength at a distance 30 cm from the four charges in the figure if $q = 1\mu$C and $a = 1$ mm?

A 18 MV/mm
B 200 V/mm
C 60 kV/m
D 18 V/m



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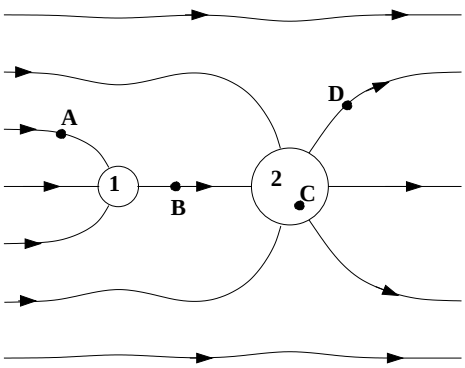
11) The figure below shows electric field lines in a region that contains two metal spheres. What can you say about the net charge on the two spheres?

A Negative on both spheres.
B Zero on sphere 1, positive on sphere 2.
C Positive on both spheres.
D Negative on sphere 1, zero on sphere 2.



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12) In the figure in question 11, in which of the four positions A, B, C and D is the potential largest?

A
B
C
D

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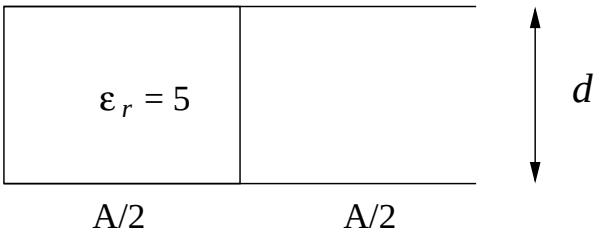
13) A parallel plate capacitor has square metal plates with area $A = a^2$, and the distance between the plates is d. The volume between the plates is partly filled with air (right half) and partly filled with a dielectric with relative permittivity $\varepsilon_r = 5$ (left half). The metal plates are large compared with the distance between them, i.e., $a \gg d$. What is the capacitance of this capacitor? ($C_0 \equiv \varepsilon_0 a^2/d$)

A $5C_0/3$
B $5C_0/6$
C $3C_0$
D $6C_0$



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14) In a region of space the electric field is

$$\boldsymbol{E}(r) = E_0 \left(\frac{r}{r_0} - \frac{r^3}{r_0^3} \right) \hat{r}$$

Here, E_0 and r_0 are constants, while r denotes the distance from the origin. How much net charge is then inside a spherical shell of radius $2r_0$, centered at the origin?

A 0 B $-96\pi\varepsilon_0 r_0^2 E_0$ C $-24\pi\varepsilon_0 r_0^2 E_0$ D $-144\pi\varepsilon_0 r_0^2 E_0$

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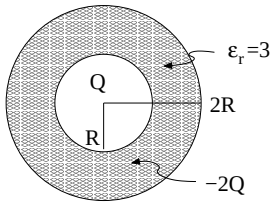
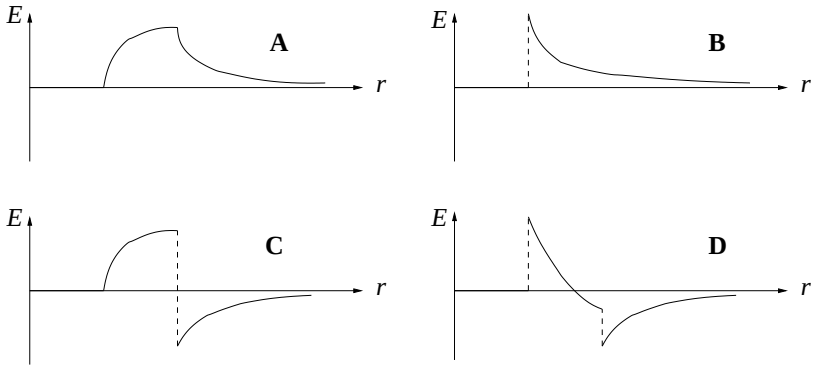
15) A metal sphere has radius R and positive charge Q. The sphere is covered with a layer of plastic (i.e.: a dielectric) with thickness R and relative permittivity $\varepsilon_r = 3$. Inside the plastic layer, a negative charge (free, but not mobile) $-2Q$ is uniformly distributed (i.e.: constant charge pr unit volume). Which of the graphs A – D shows the resulting electric field $E(r)$ (such that $\boldsymbol{E}(r) = E(r)\,\hat{r}$)?



A
B
C
D



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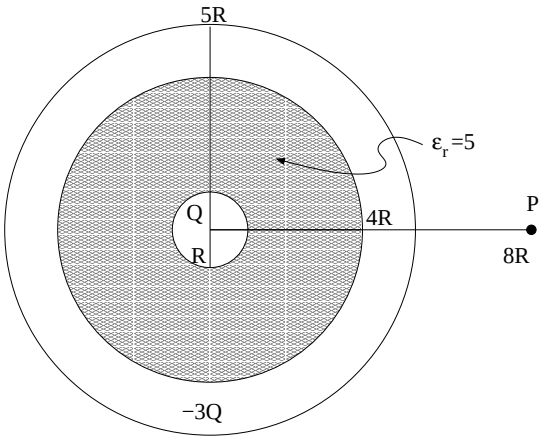
16) A metal sphere has radius R and positive charge Q. The sphere is covered with a layer of electrically neutral plastic (i.e.: dielectric) with thickness $3R$ and relative permittivity $\varepsilon_r = 5$. Outside the layer of plastic, there is a metallic spherical shell of thickness R and net charge $-3Q$. How much charge is located on the outer surface of the metallic spherical shell?

A $-3Q$
B $-2Q$
C $-Q$
D 0



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17) In question 16, what is the electric field in position P, i.e., at a distance $8R$ from the centre (the origin)?

A $-Q/128\pi\varepsilon_0 R^2$
B $-3Q/128\pi\varepsilon_0 R^2$
C 0
D $-Q/256\pi\varepsilon_0 R^2$

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18) In question 16, what is the potential difference between the innermost metal sphere and position P?

A $Q/16\pi\varepsilon_0 R^2$
B $Q/32\pi\varepsilon_0 R$
C 0
D $Q^2/4\pi\varepsilon_0 R$

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19) How much work must be done in order to change the charge from zero to $-2Q$ on a metal sphere of radius R?

A $2Q^2/\pi\varepsilon_0 R$
B $Q^2/\pi\varepsilon_0 R$
C $Q^2/2\pi\varepsilon_0 R$
D $Q^2/4\pi\varepsilon_0 R$

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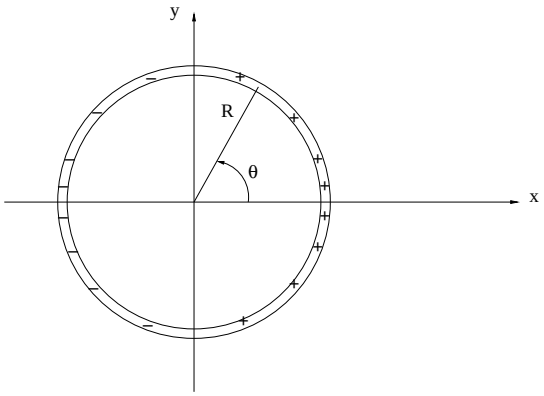
20) A thin ring of radius R has charge $\lambda(\theta) = \lambda_0 \cos\theta$ pr unit length. The ring lies in the xy plane with its center at the origin. The angle θ is as indicated in the figure below. What is the dipole moment of the ring?

A $\lambda_0 R^2/4$
B $\pi\lambda_0 R^2$
C $4\pi\lambda_0 R^2$
D $\lambda_0 R^2/\pi$



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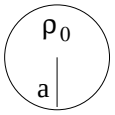
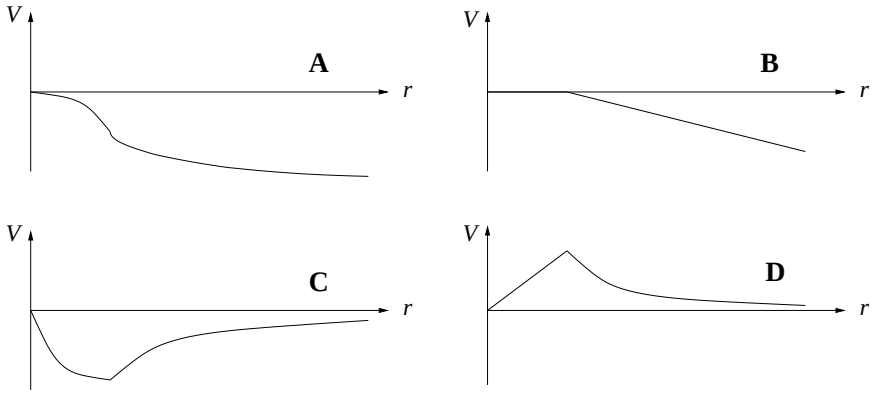
21) The figure shows a cross section of an infinitely long straight wire of radius a and with uniform charge ρ_0 pr unit volume. Which of the graphs A – D illustrates the potential V as a function of the distance r from the center axis of the wire? (Here we have chosen $V = 0$ in $r = 0$.)



A
B
C
D



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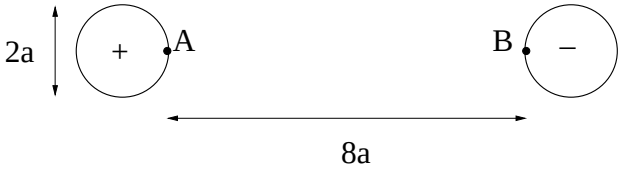
22) The figure shows a cross section through two parallel infinitely long straight wires, both with radius a. The distance between the wires (center-to-center) is $10a$. The two wires have uniform charge pr unit volume, ρ_0 and $-\rho_0$, respectively. What is then the potential difference between the two points A and B in the figure? (The distance from A to B is $8a$.)

A $\varepsilon_0\rho_0 a^2/\pi$
B $\rho_0 a^2 e^3/\varepsilon_0$
C $\varepsilon_0\rho_0 a^2 \ln 8$
D $\rho_0 a^2 \ln 9/\varepsilon_0$



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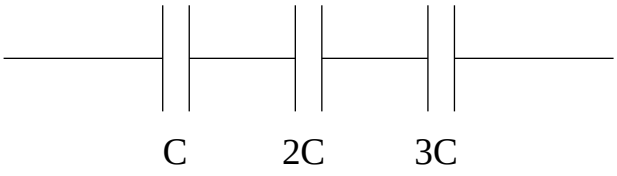
23) The figure shows three capacitors connected in series. What is the total capacitance of the system?

A $6C/11$
B $11C/6$
C $11C$
D $6C$



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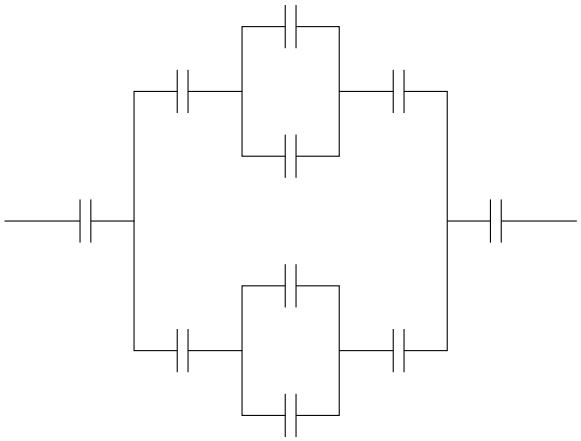
24) The figure shows ten capacitors connected together. Each of them has a capacitance C. What is the total capacitance of the system?

A $10C$
B $13C/4$
C $4C/13$
D $C/10$



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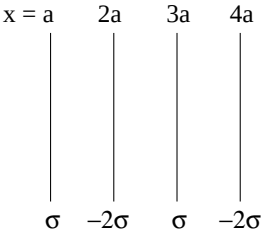
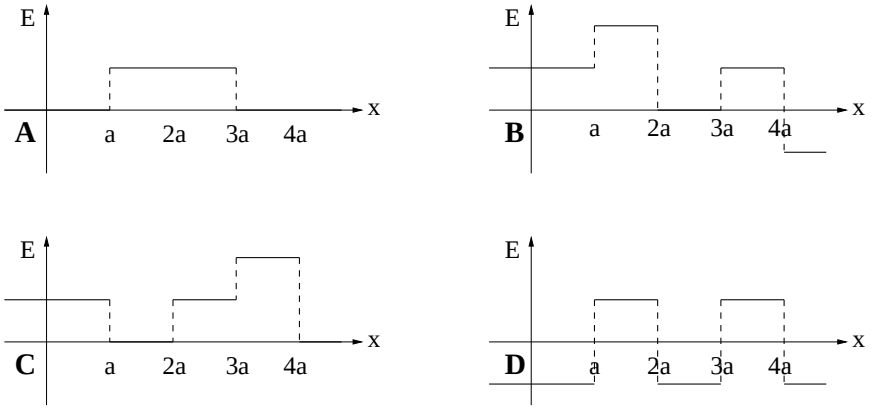
25) Four infinitely large planes are located in $x = a, 2a, 3a$ og $4a$. The four planes have charge pr unit area $\sigma, -2\sigma, \sigma$ and -2σ, respectively. Which figure shows the resulting electric field $E(x)$ (so that $\boldsymbol{E}(x) = E(x)\,\hat{x}$)?



A
B
C
D



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Institutt for fysikk, NTNU

FY1003/TFY4155 Elektrisitet og magnetisme I/Elektromagnetisme

Midterm exam Wednesday March 7 2007, 1300 – 1500.

Course code: ☐ Student number: ☐

Table for your answers – Version A

Question	A	B	C	D	Question	A	B	C	D
1	☐	☐	☐	☐	14	☐	☐	☐	☐
2	☐	☐	☐	☐	15	☐	☐	☐	☐
3	☐	☐	☐	☐	16	☐	☐	☐	☐
4	☐	☐	☐	☐	17	☐	☐	☐	☐
5	☐	☐	☐	☐	18	☐	☐	☐	☐
6	☐	☐	☐	☐	19	☐	☐	☐	☐
7	☐	☐	☐	☐	20	☐	☐	☐	☐
8	☐	☐	☐	☐	21	☐	☐	☐	☐
9	☐	☐	☐	☐	22	☐	☐	☐	☐
10	☐	☐	☐	☐	23	☐	☐	☐	☐
11	☐	☐	☐	☐	24	☐	☐	☐	☐
12	☐	☐	☐	☐	25	☐	☐	☐	☐
13	☐	☐	☐	☐					

NB: Make sure you have answered each question with exactly one answer!!!!!
NB: Make sure you have the same version (A, B, C, D or E) on the exercises and the table with your answers!!!!!