Institutt for fysikk, NTNU
FY1003 Elektrisitet og magnetisme
TFY4155 Elektromagnetisme
Spring 2008

Midterm exam Thursday March 6 2008, 1000 – 1200.

Questions on page 3 –10. Fill in the table on page 11. Mark your answers clearly.
Remember to fill in your student number on page 11.
It is sufficient to hand in the table with your answers on page 11.

Allowed remedies: C

- K. Rottmann: Matematisk formelsamling. (Or similar.)
- O. Øgrim and B. E. Lian: Størrelser og enheter i fysikk og teknikk or B. E. Lian and C. Angell: Fysiske størrelser og enheter.
- Approved calculator, with empty memory, according to list provided by NTNU. (HP30S or similar.)
- Formulas, Electrostatics is included on page 2.

Given information:

- The exam consists of 25 questions. Each question has one correct and three incorrect answers.
- Choose *one* alternative for *each* question. If you choose *more than one* alternative or *no* alternative, the answer will be considered wrong.
- Unless stated otherwise, we assume that the system is in electrostatic equilibrium.
- Unless stated otherwise, ”potential” means ”electrostatic potential”, and analogously for ”potential energy”.
- Unless stated otherwise, zero potential and potential energy is chosen at infinity.
- Metal is synonymous with electric conductor. Insulator is synonymous with dielectric. ”Large plane” is synonymous with ”approximately infinitely large plane”.
- Some fundamental constants: $\varepsilon_0 = 8.85 \cdot 10^{-12}$ C^2/Nm^2, $1/4\pi\varepsilon_0 = 9 \cdot 10^9$ Nm^2/C^2, $e = 1.6 \cdot 10^{-19}$ C, $m_e = 9.11 \cdot 10^{-31}$ kg, $m_p = 1.67 \cdot 10^{-27}$ kg, $g = 9.8$ m/s^2, $c = 3 \cdot 10^8$ m/s.
- Symbols are given in italics (e.g. V for potential) while units are given in non-italics (e.g. V for volt).
- SI-prefixes: M (mega) = 10^6, k (kilo) = 10^3, c (centi) = 10^{-2}, m (milli) = 10^{-3}, μ (micro) = 10^{-6}, n (nano) = 10^{-9}, p (pico) = 10^{-12}.
- Circumference of circle: $2\pi r$. Area of spherical surface: $4\pi r^2$. Volume of sphere: $4\pi r^3/3$.
- Gradient in cartesian coordinates: $\nabla f = (\partial f/\partial x)\,\hat{x} + (\partial f/\partial y)\,\hat{y} + (\partial f/\partial z)\,\hat{z}$
- Gradient of spherically symmetric function $f(r)$: $\nabla f = (\partial f/\partial r)\,\hat{r}$

Formulas, Electrostatics

$\int d\boldsymbol{A}$ denotes surface integral and $\int d\boldsymbol{l}$ denotes line integral. $\oint$ denotes integral over closed surface or around closed curve. **Boldface** symbols denote vectors. Symbols with a "hat" above denote unit vectors. The validity of the formulas and the meaning of the various symbols are assumed to be known.

- Coulomb's law:
$$\boldsymbol{F} = \frac{qq'}{4\pi\varepsilon_0 r^2}\hat{r}$$
- Electric field and potential:
$$\boldsymbol{E} = -\nabla V$$
$$\Delta V = V_B - V_A = -\int_A^B \boldsymbol{E}\cdot d\boldsymbol{l}$$
- Electric potential from point charge:
$$V = \frac{q}{4\pi\varepsilon_0 r}$$
- Electric flux:
$$\phi_E = \int \boldsymbol{E}\cdot d\boldsymbol{A}$$
- Electrostatic force is conservative:
$$\oint \boldsymbol{E}\cdot d\boldsymbol{l} = 0$$
- Gauss' law for electric field and electric displacement:
$$\varepsilon_0 \oint \boldsymbol{E}\cdot d\boldsymbol{A} = q$$
$$\oint \boldsymbol{D}\cdot d\boldsymbol{A} = q_{\text{free}}$$
- Electric displacement:
$$\boldsymbol{D} \equiv \varepsilon_0\boldsymbol{E} + \boldsymbol{P} = \varepsilon_r\varepsilon_0\boldsymbol{E} = \varepsilon\boldsymbol{E}$$
- Electric dipole moment; in general, for region Ω with distribution of charge:
$$\boldsymbol{p} = \int_\Omega \boldsymbol{r}\, dq$$
- Electric dipole moment; for point charges $\pm q$ separated by distance $\boldsymbol{d}$:
$$\boldsymbol{p} = q\boldsymbol{d}$$
- Electric polarization = electric dipole moment pr unit volume:
$$\boldsymbol{P} = \frac{\Delta\boldsymbol{p}}{\Delta V}$$
  Linear response:
$$\boldsymbol{P} = \varepsilon_0\chi_e\boldsymbol{E}$$
- Capacitance:
$$C = \frac{q}{V}$$
- Energy density (energy pr unit volume) in electric field:
$$u_E = \frac{1}{2}\varepsilon_0 E^2$$

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1) Which statement is wrong?

A The potential on a conductor is constant.
B The unit F/m may be used for electric permittivity.
C The capacitance of a capacitor increases linearly with the charge on the capacitor.
D Two point charges $2Q$ and $2Q$ separated by a distance $2R$ has twice as much potential energy as two point charges $-Q$ and $-Q$ separated by a distance R.

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2) Which statement is correct?

A Electric dipole moment may be measured in the unit C/m.
B Electric polarization may be measured in the unit C/m.
C Electric dipole moment may be measured in the unit C/m^2.
D Electric polarization may be measured in the unit C/m^2.

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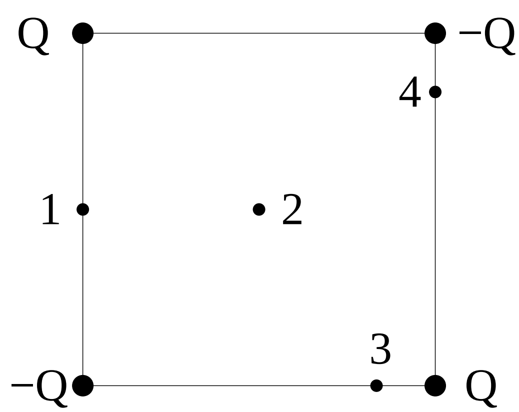
3) Four point charges, two positive (Q) and two negative ($-Q$), are located in the corners of a square. Rank the electric potential in the four points 1, 2, 3 and 4. (Point 1 is at the center of the left "edge", point 2 is at the center of the square.)

A $V_1 > V_3 > V_4 > V_2$
B $V_1 = V_2 = V_3 = V_4$
C $V_3 > V_1 = V_2 > V_4$
D $V_1 > V_3 = V_4 > V_2$



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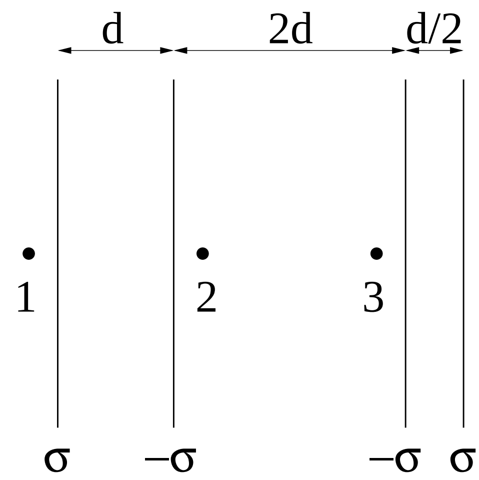
4) Four large parallel planes are separated by distances d, $2d$ and $d/2$, as shown in the figure. The planes have charge pr unit area σ, $-\sigma$, $-\sigma$ and σ (from left to right, with $\sigma > 0$). Rank the electric potential in the four points marked with 1, 2, 3 and 4.

A $V_1 > V_2 > V_3 > V_4$
B $V_1 > V_4 > V_2 = V_3$
C $V_4 = V_1 > V_2 = V_3$
D $V_1 = V_4 > V_3 > V_2$



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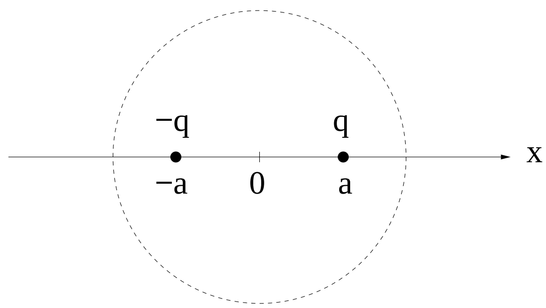
5) Two point charges $\pm q$ are located in $x = \pm a$ $(y = z = 0)$. What is the net electric flux through a spherical surface with radius $3a/2$ and center at the origin?

A $-q/\varepsilon_0$
B 0
C q/ε_0
D $2q/\varepsilon_0$



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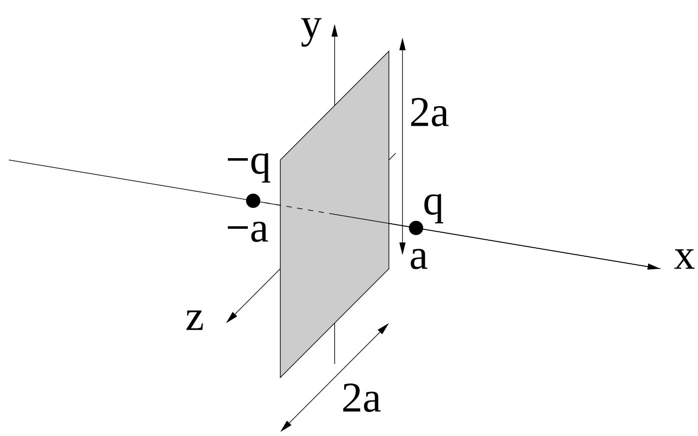
6) Two point charges $\pm q$ are located in $x = \pm a$ $(y = z = 0)$. What is the net electric flux through the part of the yz plane limited by $-a \le y \le a$, $-a \le z \le a$ (and $x = 0$)?

A 0
B $q/6\varepsilon_0$
C $q/4\varepsilon_0$
D $q/3\varepsilon_0$



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7) The electric field in a region is

$$\boldsymbol{E}(x,z) = \hat{x}\, E_0 \cos kx - \hat{z}\, E_0 \sin kz,$$

where k and E_0 are constants. What is the potential difference between the origin and the point $(\pi/k, \pi/k, \pi/k)$, i.e.,

$$\Delta V = V(\frac{\pi}{k}, \frac{\pi}{k}, \frac{\pi}{k}) - V(0,0,0)?$$

A $\Delta V = 0$
B $\Delta V = E_0/2k$
C $\Delta V = E_0/k$
D $\Delta V = 2E_0/k$

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8) The potential in a region is

$$V(y) = k\, V_0\, y,$$

where k and V_0 are constants. How much potential electric energy U_E is then stored in the volume limited by

$$0 \le x \le \pi/k \quad , \quad 0 \le y \le \pi/k \quad , \quad 0 \le z \le \pi/k \ ?$$

A $U_E = \varepsilon_0 V_0^2 \pi^3/2k$
B $U_E = \varepsilon_0 V_0^2 \pi^2/4k$
C $U_E = \varepsilon_0 V_0^4 \pi/k$
D $U_E = 0$

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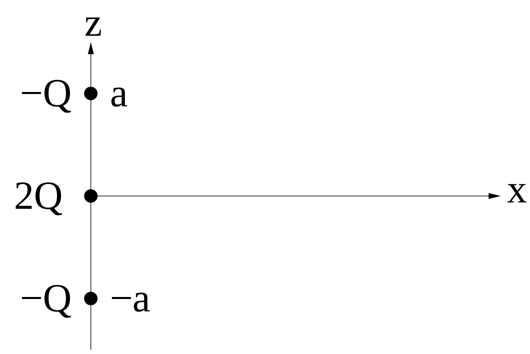
9) The figure shows a system with 3 point charges located on the z axis: $-Q$ in $z = -a$ and in $z = a$, and $2Q$ in $z = 0$. What is the electric dipole moment $\boldsymbol{p}$ of this system?

A $\boldsymbol{p} = 0$
B $\boldsymbol{p} = Qa\,\hat{z}$
C $\boldsymbol{p} = 2Qa\,\hat{z}$
D $\boldsymbol{p} = -Qa\,\hat{z}$



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10) What is the electric field $\boldsymbol{E}(x)$ on the positive x axis in question 9?

A $\boldsymbol{E}(x) = \frac{Q\hat{x}}{2\pi\varepsilon_0}\left[\frac{1}{x^2} + \frac{1}{x^2+a^2}\right]$

B $\boldsymbol{E}(x) = \frac{Q\hat{x}}{2\pi\varepsilon_0}\left[\frac{1}{x^2} + \frac{x}{(x^2+a^2)^{3/2}}\right]$

C $\boldsymbol{E}(x) = \frac{Q\hat{x}}{2\pi\varepsilon_0}\left[\frac{1}{x^2} - \frac{1}{x^2+a^2}\right]$

D $\boldsymbol{E}(x) = \frac{Q\hat{x}}{2\pi\varepsilon_0}\left[\frac{1}{x^2} - \frac{x}{(x^2+a^2)^{3/2}}\right]$

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11) Far out on the positive x axis (i.e. $x \gg a$), the potential $V(x)$ in question 9 may approximately be expressed as

A $\quad V(x) \simeq \dfrac{Q}{4\pi\varepsilon_0 x}$ $\qquad$ B $\quad V(x) \simeq \dfrac{Qa}{4\pi\varepsilon_0 x^2}$

C $\quad V(x) \simeq \dfrac{Qa^2}{4\pi\varepsilon_0 x^3}$ $\qquad$ D $\quad V(x) \simeq \dfrac{Qx}{4\pi\varepsilon_0 a^2}$

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12) The system in question 9 has potential energy

A $\quad U = -\dfrac{15Q^2}{16\pi\varepsilon_0 a}$ $\qquad$ B $\quad U = -\dfrac{15Q^2}{16\pi\varepsilon_0 a^2}$

C $\quad U = -\dfrac{7Q^2}{8\pi\varepsilon_0 a^2}$ $\qquad$ D $\quad U = -\dfrac{7Q^2}{8\pi\varepsilon_0 a}$

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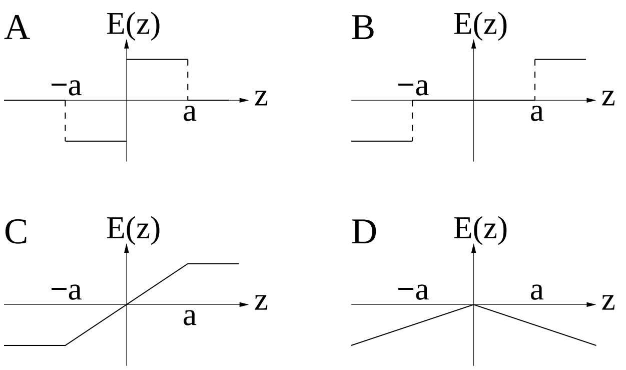
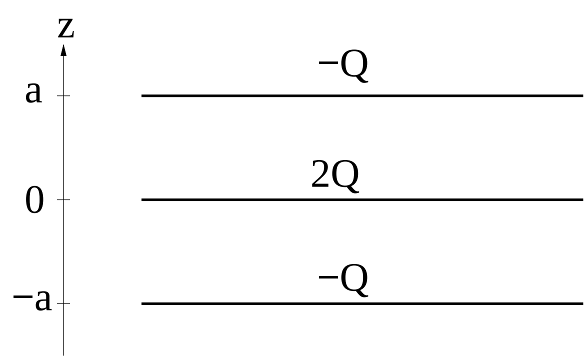
13) Three thin uniformly charged large parallel metal plates with area A and charge $-Q, 2Q, -Q$ are located in $z = -a, 0 - a$, as shown in the right figure. Which graph shows correctly the resulting electric field $E(z)$ (so that $\boldsymbol{E}(z) = E(z)\,\hat{z}$)?





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14) What is the total potential energy of the three charged plates in question 13?

A $\quad U = \dfrac{Q^2}{\varepsilon_0 a}$ $\qquad$ B $\quad U = \dfrac{Q^2}{\varepsilon_0 A}$

C $\quad U = \dfrac{Q^2 A}{\varepsilon_0 a}$ $\qquad$ D $\quad U = \dfrac{Q^2 a}{\varepsilon_0 A}$

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15) For the system in question 13, the potential is set to zero on the lower plate, i.e., $V(-a) = 0$. What is now the potentials $V(0)$ and $V(a)$ on the central and upper plates, respectively?

A $V(0) = \frac{Qa}{\varepsilon_0 A}$, $V(a) = 0$

B $V(0) = \frac{Qa}{\varepsilon_0 A}$, $V(a) = \frac{2Qa}{\varepsilon_0 A}$

C $V(0) = -\frac{Qa}{\varepsilon_0 A}$, $V(a) = 0$

D $V(0) = -\frac{Qa}{\varepsilon_0 A}$, $V(a) = -\frac{2Qa}{\varepsilon_0 A}$

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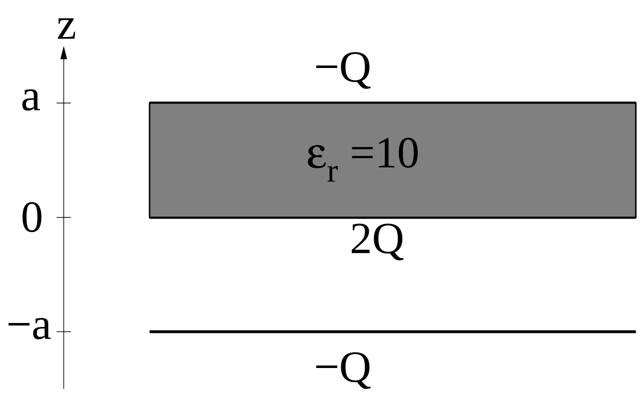
16) In the system in question 13, the volume between the central and upper plate is filled with a dielectric slab with relative permittivity $\varepsilon_r = 10$. What is now the potential difference $\Delta V = V(a) - V(-a)$ between the upper and the lower plate?

A $\Delta V = \frac{Qa}{10\varepsilon_0 A}$

B $\Delta V = \frac{9Qa}{10\varepsilon_0 A}$

C $\Delta V = \frac{10Qa}{\varepsilon_0 A}$

D $\Delta V = \frac{10Qa}{9\varepsilon_0 A}$



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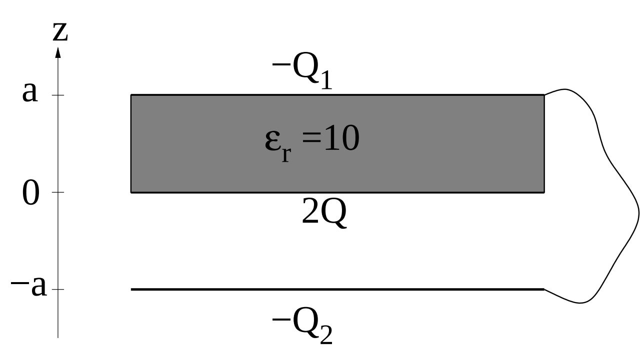
17) The upper and lower plate of question 16 are connected with a thin conducting wire, so that the two plates obtain equal electric potential. Find the resulting charges $-Q_1$ and $-Q_2$ on the upper and lower plate, respectively. (You may assume that the thin wire connecting the two plates is always electrically neutral.)

A $-Q_1 = -20Q/11$, $-Q_2 = -2Q/11$
B $-Q_1 = -2Q/11$, $-Q_2 = -20Q/11$
C $-Q_1 = -20Q/9$, $-Q_2 = 2Q/9$
D $-Q_1 = 2Q/9$, $-Q_2 = -20Q/9$



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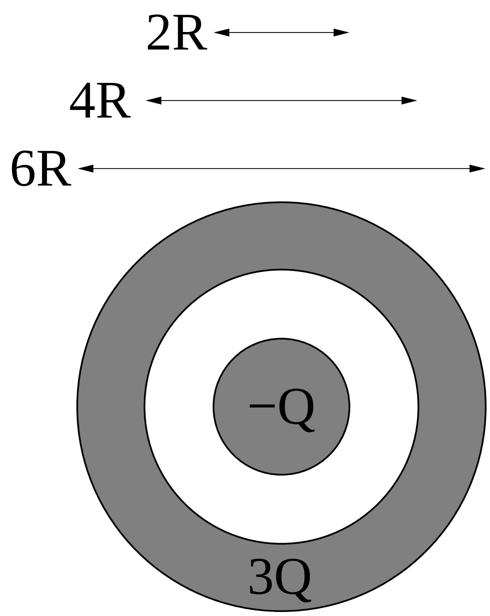
18) A metal sphere with radius R and (negative) charge $-Q$ er is surrounded by a vacuum layer of thickness R, followed by a metallic spherical shell of thickness R and charge $3Q$. How much charge is located on the outer surface of the metallic spherical shell?



A 0
B Q
C $2Q$
D $3Q$

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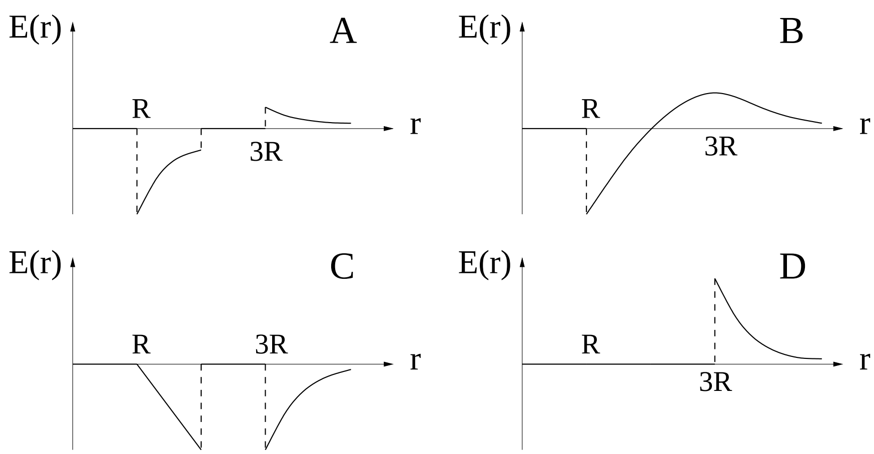
19) Which graph illustrates the correct $E(r)$ (so that $\boldsymbol{E}(r) = E(r)\,\hat{r}$) for the system in question 18?



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20) What is the potential difference between the spherical shell and a point at a distance $6R$ from the center of the system in question 18, i.e., $\Delta V = V(r = 3R) - V(r = 6R)$?

A $\Delta V = Q/3\pi\varepsilon_0 R$
B $\Delta V = Q/6\pi\varepsilon_0 R$
C $\Delta V = Q/12\pi\varepsilon_0 R$
D $\Delta V = Q/24\pi\varepsilon_0 R$

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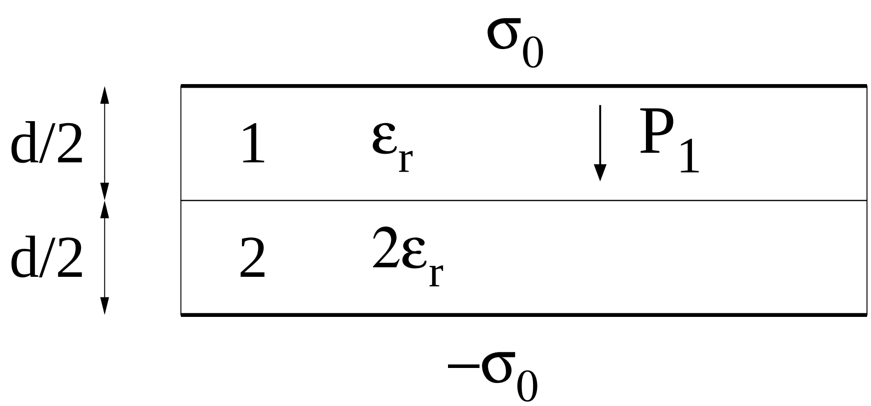
21) Two large metallic planes have area A and charge pr unit area σ_0 (upper plate) and $-\sigma_0$ (lower plate), respectively. The distance between the plates is d. The volume between the metal plates is filled with two dielectric slabs. Medium 1, in the upper half, has relative permittivity ε_r, whereas medium 2, in the lower half, has relative permittivity $2\varepsilon_r$. What is the polarization P_1 in medium 1?

A $P_1 = \left(1 - \dfrac{1}{\varepsilon_r}\right) \sigma_0$

B $P_1 = \varepsilon_r \, \sigma_0$

C $P_1 = \left(1 + \dfrac{1}{\varepsilon_r}\right) \sigma_0$

D $P_1 = \dfrac{\sigma_0}{\varepsilon_r}$



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22) What is the capacitance C of the parallel plate capacitor in question 21? (Hint: This may be regarded as two capacitors connected in series.)

A $C = 3\varepsilon_r\varepsilon_0 A/2d$
B $C = 4\varepsilon_r\varepsilon_0 A/3d$
C $C = 5\varepsilon_r\varepsilon_0 A/4d$
D $C = 6\varepsilon_r\varepsilon_0 A/5d$

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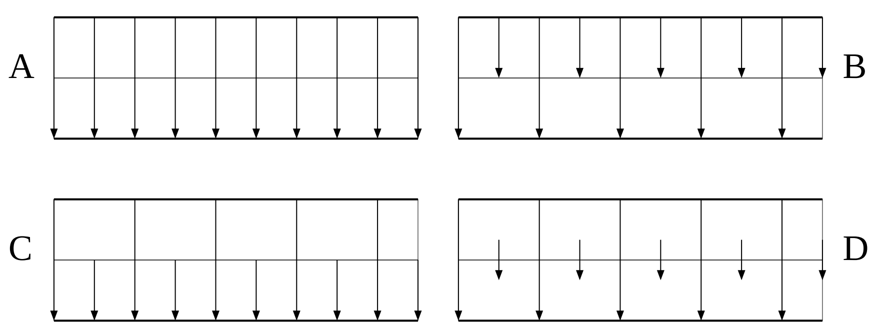
23) Which of the figures below illustrates field lines for the electric field $\boldsymbol{E}$ in the parallel plate capacitor in question 21?



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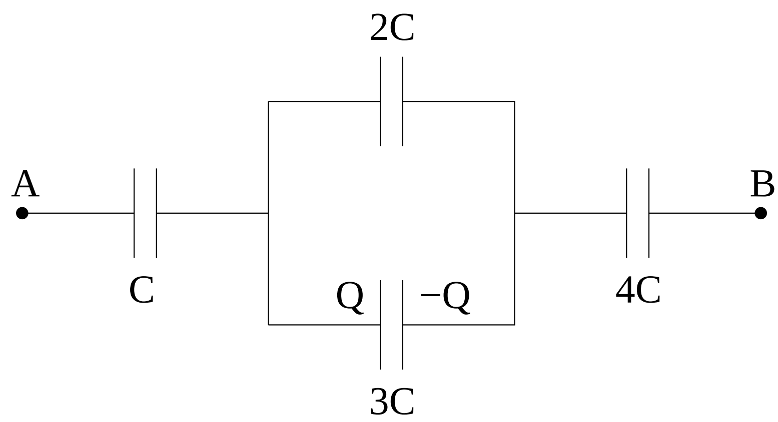
24) The figure shows a connection of 4 capacitances, C, $2C$, $3C$ and $4C$. What is the total capacitance of this system?



A $29C/5$
B $10C$
C $20C/29$
D $15C/2$

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25) Suppose there is a potential difference $V_0 = V_A - V_B$ between the two end points A and B in question 24. Then, how much charge $(\pm)Q$ is on the capacitance $3C$?

A $Q = 29V_0C/12$
B $Q = 31V_0C/5$
C $Q = 5V_0C/31$
D $Q = 12V_0C/29$

FY1003/TFY4155 Elektrisitet og magnetisme/Elektromagnetisme

Midterm exam Thursday March 6 2008, 1000 – 1200.

Course code: ☐ Student number: ☐

Table for your answers

Question	A	B	C	D	Question	A	B	C	D
1	☐	☐	☐	☐	14	☐	☐	☐	☐
2	☐	☐	☐	☐	15	☐	☐	☐	☐
3	☐	☐	☐	☐	16	☐	☐	☐	☐
4	☐	☐	☐	☐	17	☐	☐	☐	☐
5	☐	☐	☐	☐	18	☐	☐	☐	☐
6	☐	☐	☐	☐	19	☐	☐	☐	☐
7	☐	☐	☐	☐	20	☐	☐	☐	☐
8	☐	☐	☐	☐	21	☐	☐	☐	☐
9	☐	☐	☐	☐	22	☐	☐	☐	☐
10	☐	☐	☐	☐	23	☐	☐	☐	☐
11	☐	☐	☐	☐	24	☐	☐	☐	☐
12	☐	☐	☐	☐	25	☐	☐	☐	☐
13	☐	☐	☐	☐					

NB: Make sure you have answered each question with exactly one answer.