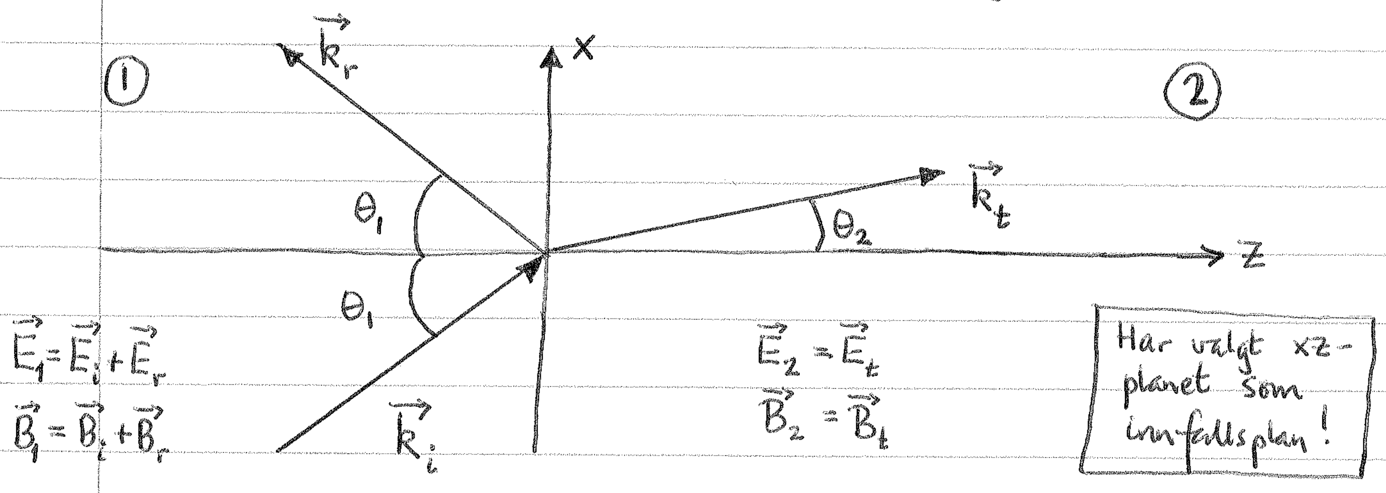
Ekstratimen 25.10.06

100A

Litt mer om refl. & transm. av e.m. bølger



$$\vec{E}_i(\vec{r},t) = \vec{E}_{i0}\cos(\vec{k}_i\cdot\vec{r} - \omega t) \qquad \vec{B}_i = \frac{1}{\omega}\vec{k}_i \times \vec{E}_i$$

$$\vec{E}_r(\vec{r},t) = \vec{E}_{r0}\cos(\vec{k}_r\cdot\vec{r} - \omega t) \qquad \vec{B}_r = \frac{1}{\omega}\vec{k}_r \times \vec{E}_r$$

$$\vec{E}_t(\vec{r},t) = \vec{E}_{t0}\cos(\vec{k}_t\cdot\vec{r} - \omega t) \qquad \vec{B}_t = \frac{1}{\omega}\vec{k}_t \times \vec{E}_t$$

$$k_i v_1 = k_r v_1 = k_t v_2 = \omega\,, \quad v_1 = \frac{c}{n_1}\,, \quad v_2 = \frac{c}{n_2}$$

Fra Maxwells ligninger:
$$\left.\begin{aligned} \varepsilon_1 E_{1z} &= \varepsilon_2 E_{2z} \\ B_{1z} &= B_{2z} \\ E_{1x} &= E_{2x} \\ E_{1y} &= E_{2y} \\ \frac{1}{\mu_1}B_{1x} &= \frac{1}{\mu_2}B_{2x} \\ \frac{1}{\mu_1}B_{1y} &= \frac{1}{\mu_2}B_{2y} \end{aligned}\right\} \text{ i grenseflaten } z=0$$

F.eks: $\varepsilon_1 E_{i0z}\cos(\vec{k}_i\cdot\vec{r} - \omega t) + \varepsilon_1 E_{r0z}\cos(\vec{k}_r\cdot\vec{r} - \omega t) = \varepsilon_2 E_{t0z}\cos(\vec{k}_t\cdot\vec{r} - \omega t)$

Øving 9 $\Rightarrow \vec{k}_i\cdot\vec{r} = \vec{k}_r\cdot\vec{r} = \vec{k}_t\cdot\vec{r} \Rightarrow$ alle cos(...) kan forkortes

$$\Rightarrow \varepsilon_1 E_{i0z} + \varepsilon_1 E_{r0z} = \varepsilon_2 E_{t0z} \qquad \text{osv.}$$

$$\Rightarrow \varepsilon_1 (E_{ioz} + E_{roz}) = \varepsilon_2 E_{toz}$$

$$B_{ioz} + B_{roz} = B_{toz}$$

$$E_{iox} + E_{rox} = E_{tox}$$

$$E_{ioy} + E_{roy} = E_{toy}$$

$$\frac{1}{\mu_1}(B_{iox} + B_{rox}) = \frac{1}{\mu_2} B_{tox}$$

$$\frac{1}{\mu_1}(B_{ioy} + B_{roy}) = \frac{1}{\mu_2} B_{toy}$$

Generell innkommende bølge:

$$\vec{E}_i(\vec{r},t) = \vec{E}_{io} \cos(\vec{k}_i \cdot \vec{r} - \omega t)$$

$$= [E_{iox}\hat{x} + E_{ioy}\hat{y} + E_{ioz}\hat{z}] \cos(\vec{k}_i \cdot \vec{r} - \omega t)$$

kan dekomponeres i $\vec{E}_{ip}$ parallelt med innfallsplanet og $\vec{E}_{in}$ normalt på innfallsplanet:

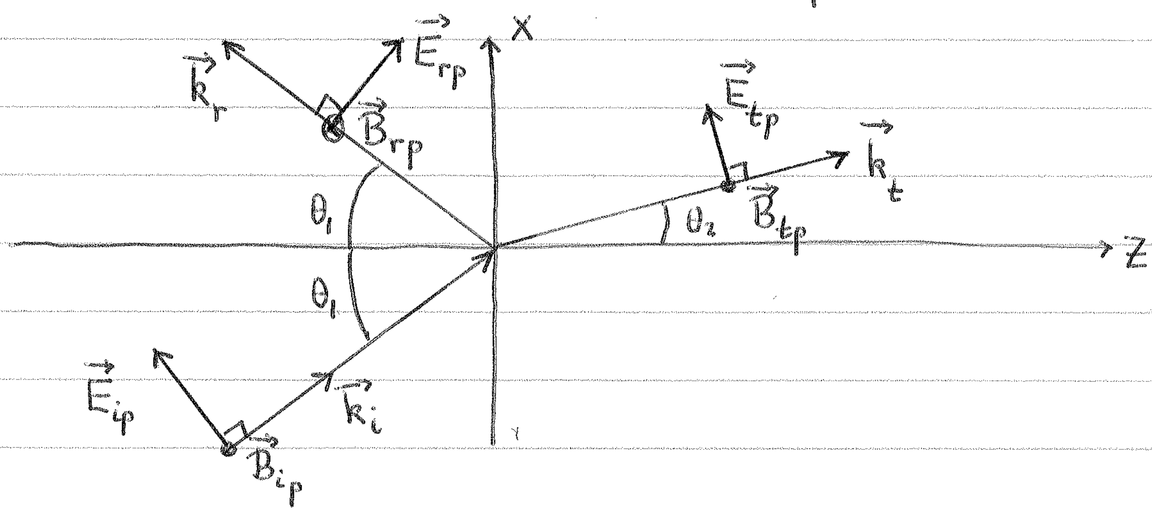
$$\vec{E}_{ip} = [E_{iox}\hat{x} + E_{ioz}\hat{z}] \cos(\vec{k}_i \cdot \vec{r} - \omega t)$$

$$\vec{E}_{in} = E_{ioy}\hat{y} \cos(\vec{k}_i \cdot \vec{r} - \omega t)$$

$$\vec{E}_i = \vec{E}_{ip} + \vec{E}_{in}$$

100c

Bølge polarisert <u>parallelt</u> med innfallsplanet:



$E_{ipox} = E_{ipo} \cos\theta_1 \; ; \; E_{ipoy} = 0 \; ; \; E_{ipoz} = -E_{ipo} \sin\theta_1$

$B_{ipox} = B_{ipoz} = 0 \quad ; \quad B_{ipoy} = B_{ipo} = \dfrac{E_{ipo}}{v_1}$

$E_{rpox} = E_{rpo} \cos\theta_1 \quad ; \quad E_{rpoy} = 0 \quad ; \quad E_{rpoz} = E_{rpo} \sin\theta_1$

$B_{rpox} = B_{rpoz} = 0 \quad ; \quad B_{rpoy} = B_{rpo} = -\dfrac{E_{rpo}}{v_1}$

$E_{tpox} = E_{tpo} \cos\theta_2 \; ; \; E_{tpoy} = 0 \quad ; \quad E_{tpoz} = -E_{tpo} \sin\theta_2$

$B_{tpox} = B_{tpoz} = 0 \quad ; \quad B_{tpoy} = B_{tpo} = \dfrac{E_{tpo}}{v_2}$

Grenseflatebetingelsene gir:

$$\varepsilon_1 \sin\theta_1 \left(-E_{ipo} + E_{rpo}\right) = -\varepsilon_2 E_{tpo} \sin\theta_2 \qquad (a)$$

$$\cos\theta_1 \left(E_{ipo} + E_{rpo}\right) = E_{tpo} \cos\theta_2 \qquad (b)$$

$$\frac{1}{\mu_1 v_1}\left(E_{ipo} - E_{rpo}\right) = \frac{1}{\mu_2 v_2} E_{tpo} \qquad (c)$$

3 ligninger med 2 ukjente ?? Nei:

Multipliser (a) med $-c/n_2$ på begge sider og bruk

$$\varepsilon_1 \sin\theta_1 = \frac{1}{v_1^2 \mu_1} \cdot \frac{n_2 \sin\theta_2}{n_1} = \frac{n_1}{v_1 c \mu_1} \cdot \frac{n_2 \sin\theta_2}{n_1}$$ og

$$\varepsilon_2 \sin\theta_2 = \frac{1}{v_2^2 \mu_2} \sin\theta_2 = \frac{n_2 \sin\theta_2}{v_2 c \mu_2} .$$ Vi ser da at

ligning (a) og (c) gir akkurat det samme, nemlig:

$$E_{ipo} - E_{rpo} = \beta E_{tpo} \qquad \left(\beta \equiv \mu_1 v_1 / \mu_2 v_2\right)$$

Mens (b) kan skrives slik:

$$E_{ipo} + E_{rpo} = \alpha E_{tpo} \qquad \left(\alpha \equiv \frac{\cos\theta_2}{\cos\theta_1}\right)$$

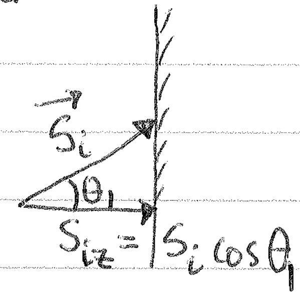
Løsning:

$$\boxed{\begin{aligned} E_{rpo} &= \frac{\alpha-\beta}{\alpha+\beta} E_{ipo} \\ E_{tpo} &= \frac{2}{\alpha+\beta} E_{ipo} \end{aligned}}$$

$$\alpha = \frac{\cos\theta_2}{\cos\theta_1} = \frac{\sqrt{1-\sin^2\theta_2}}{\cos\theta_1} = \frac{\sqrt{1 - n_1^2 \sin^2\theta_1 / n_2^2}}{\cos\theta_1}$$

[Kontroll: $\theta_1 = 0 \Rightarrow \alpha = 1 \Rightarrow E_{rpo} = \frac{1-\beta}{1+\beta} E_{ipo}$; OK !]

Intensitet: $S_z = \vec{S}\cdot\hat{z}$ = effekt pr flateenhet som treffer grenseflaten



$$\Rightarrow I_{ip} = \frac{1}{2}\varepsilon_1 v_1 E_{ipo}^2 \cdot \cos\theta_1$$

$$I_{rp} = \frac{1}{2}\varepsilon_1 v_1 E_{rpo}^2 \cdot \cos\theta_1$$

$$I_{tp} = \frac{1}{2}\varepsilon_2 v_2 E_{tpo}^2 \cdot \cos\theta_2$$

$$\Rightarrow R_p = \frac{I_{rp}}{I_{ip}} = \left(\frac{\alpha-\beta}{\alpha+\beta}\right)^2$$

$$T_p = \frac{I_{tp}}{I_{ip}} = \frac{\overbrace{\varepsilon_2 v_2}\,\overbrace{\cos\theta_2}}{\underbrace{\varepsilon_1 v_1}_{\beta}\,\underbrace{\cos\theta_1}_{\alpha}}\left(\frac{E_{tpo}}{E_{ipo}}\right)^2 = \alpha\beta\left(\frac{2}{\alpha+\beta}\right)^2$$

Brewsters vinkel: Når $\alpha=\beta$, er $R_p=0$ og $T_p=1$ for bølge med $\vec{E}$ i innfallsplanet. Det skjer når

$$\alpha^2 = \frac{1-n_1^2\sin^2\theta_B/n_2^2}{\cos^2\theta_B} = \beta^2$$

$$\Rightarrow \sin^2\theta_B = \frac{1-\beta^2}{(n_1/n_2)^2-\beta^2}$$

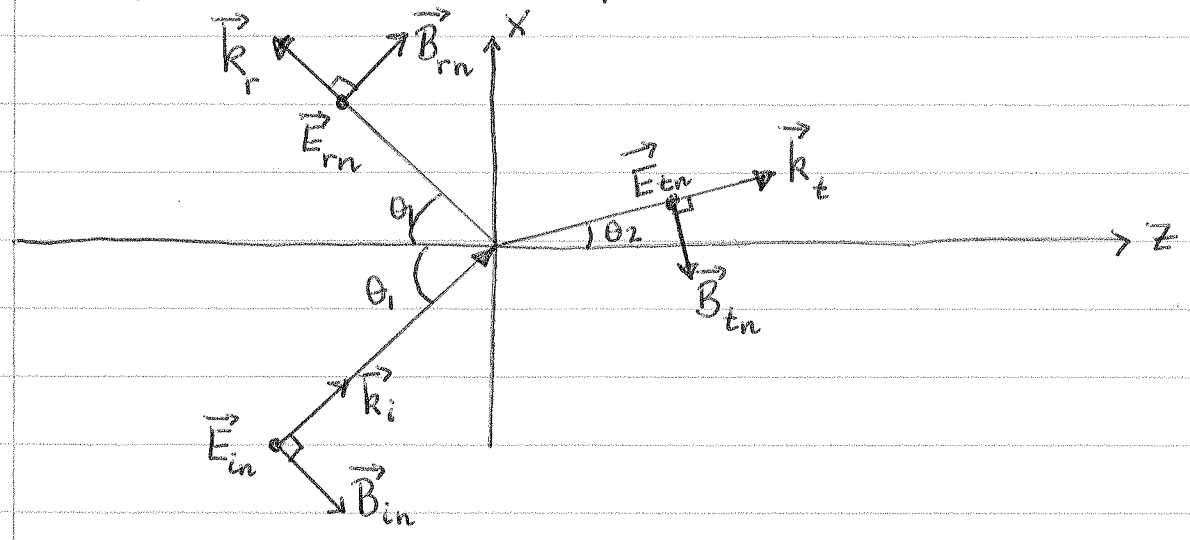
Umagnetiske medier $\Rightarrow \beta^2 \approx n_2^2/n_1^2 \Rightarrow \sin^2\theta_B \approx \dfrac{\beta^2}{1+\beta^2}$

$$\Rightarrow \boxed{\tan\theta_B \approx n_2/n_1}$$

Eks: Luft/glass $\Rightarrow \tan\theta_B \approx 1.5/1.0$

$$\Rightarrow \theta_B \approx 56^\circ$$

Bølge polarisert normalt på innfallsplanet:



$$E_{inox} = E_{inoz} = E_{rnox} = E_{rnoz} = E_{tnox} = E_{tnoz} = 0$$

$$E_{inoy} = E_{ino} \; ; \; E_{rnoy} = E_{rno} \; ; \; E_{tnoy} = E_{tno}$$

$$B_{inoy} = B_{rnoy} = B_{tnoy} = 0$$

$$B_{inox} = -B_{ino} \cos\theta_1 \; ; \; B_{inoz} = B_{ino} \sin\theta_1$$

$$B_{rnox} = B_{rno} \cos\theta_1 \; ; \; B_{rnoz} = B_{rno} \sin\theta_1$$

$$B_{tnox} = -B_{tno} \cos\theta_2 \; ; \; B_{tnoz} = B_{tno} \sin\theta_2$$

Grenseflatebetingelsene gir nå:

$$(B_{ino} + B_{rno}) \sin\theta_1 = B_{tno} \sin\theta_2 \qquad (a)$$

$$E_{ino} + E_{rno} = E_{tno} \qquad (b)$$

$$\frac{1}{\mu_1}(B_{ino} - B_{rno}) \cos\theta_1 = \frac{1}{\mu_2} B_{tno} \cos\theta_2 \qquad (c)$$

Her er det (a) og (b) som sier samme sak:

$$\left(\frac{E_{ino}}{v_1} + \frac{E_{rno}}{v_1}\right) \cdot \frac{v_1}{v_2} \sin\theta_2 = \frac{E_{tno}}{v_2} \sin\theta_2$$

$$\Rightarrow \quad E_{ino} + E_{rno} = E_{tno} \quad \text{som er (b)}$$

Lign. (c) gir (igjen med $\alpha = \frac{\cos\theta_2}{\cos\theta_1}$ og $\beta = \mu_1 v_1 / \mu_2 v_2$):

$$E_{ino} - E_{rno} = \alpha\beta \, E_{tno}$$

Løsning:

$$\boxed{\begin{aligned} E_{rno} &= \frac{1-\alpha\beta}{1+\alpha\beta} E_{ino} \\ E_{tno} &= \frac{2}{1+\alpha\beta} E_{ino} \end{aligned}}$$

$$\left.\begin{aligned} R_n &= \frac{I_{rn}}{I_{in}} = \left(\frac{E_{rno}}{E_{ino}}\right)^2 = \left(\frac{1-\alpha\beta}{1+\alpha\beta}\right)^2 \\ T_n &= \frac{I_{tn}}{I_{in}} = \beta\alpha \left(\frac{E_{tno}}{E_{ino}}\right)^2 = \alpha\beta\left(\frac{2}{1+\alpha\beta}\right)^2 \end{aligned}\right\} R_n + T_n = 1$$

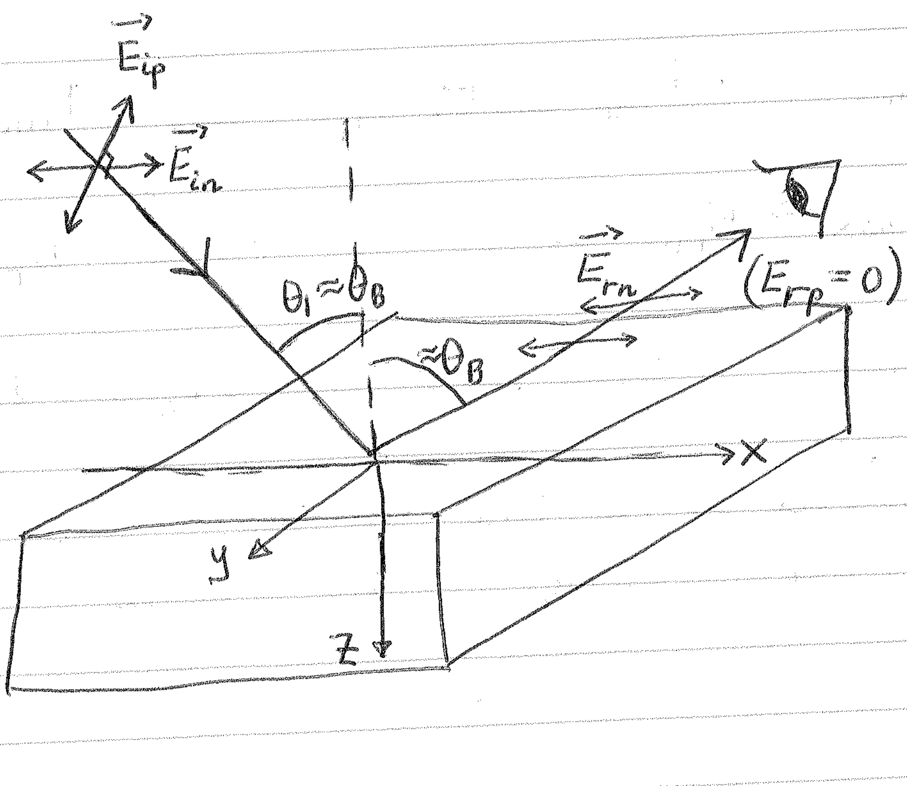
Vil "aldri" ha $\alpha\beta = 1 \Rightarrow$ Har "aldri" $R_n = 0$

Anta <u>upolarisert</u> lys inn med $\theta_1 \approx \theta_B$ = Brewsters vinkel.

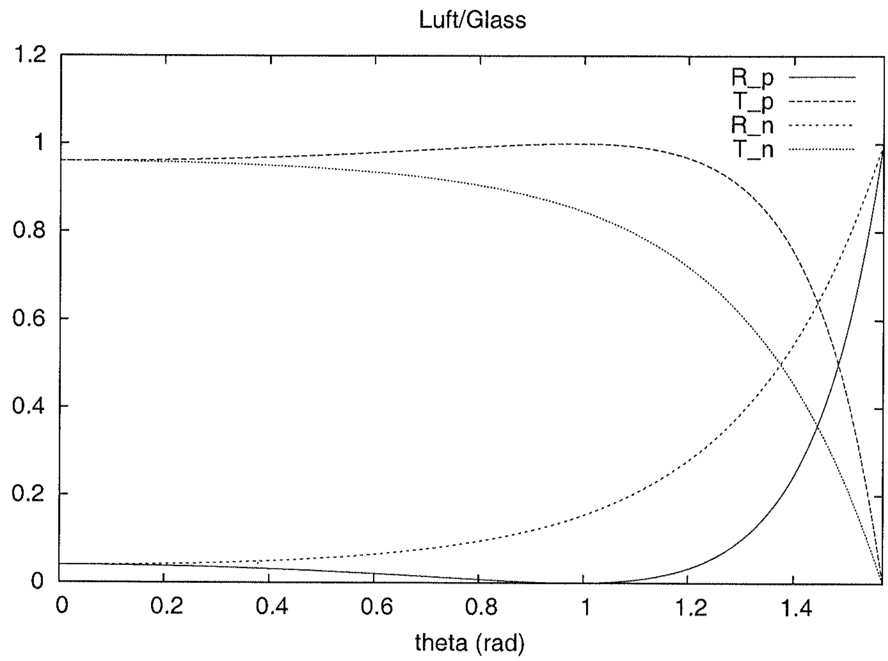
Da blir $R_p = 0$ og $R_n \neq 0$

$\Rightarrow$ Reflektert lys blir polarisert "horisontalt" (se Lekelab!)

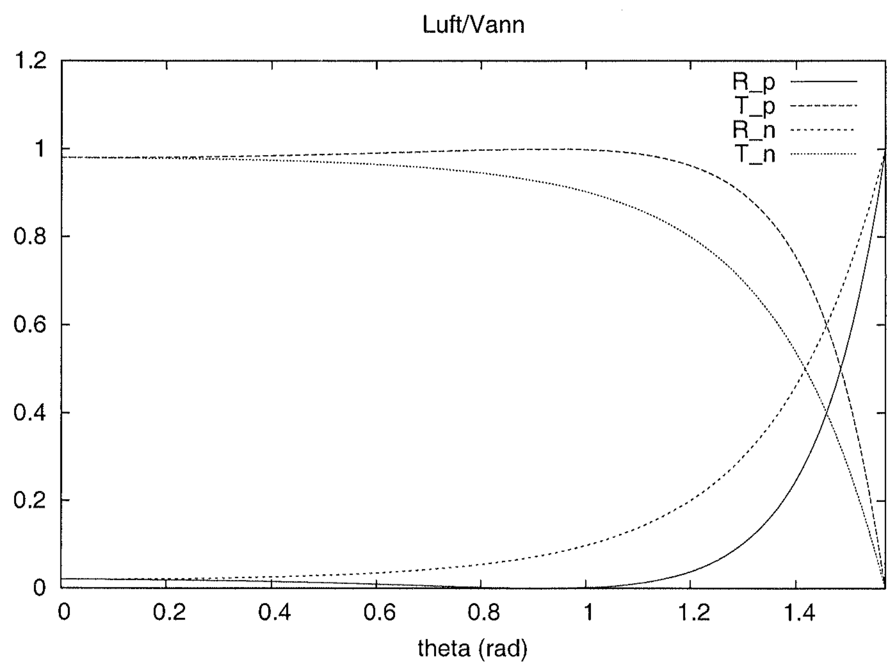
100G'



$\vec{E}_{ip}$
$\vec{E}_{in}$
$\theta_1 \approx \theta_B$
$\approx \theta_B$
$\vec{E}_{rn}$
$(E_{rp} = 0)$
x
y
z
z=0



n_2 = 1.5
beta = 1.5
theta_B = 56 grader



n_2 = 1.33
beta = 1.33
theta_B = 53 grader