Midterm Bølgefysikk 10.10.2008. LF (Versjon 0)

① $x = x_0 \sin \omega t$, $v = \omega x_0 \cos \omega t$, $a = -\omega^2 x_0 \sin \omega t$

$\Rightarrow (v\omega)^2 + a^2 = (\omega^2 x_0)^2 \Rightarrow x_0^2 = v^2/\omega^2 + a^2/\omega^4 \Rightarrow$ (D)

② $\lambda = 80\,m$; $v = \frac{\Delta x}{\Delta t} = \frac{20\,m + N \cdot 80\,m}{50\,s} = (4 + 16N)\,m/s$

$= \ldots, -60, (-44), -28, -12, 4, 20, 36, 52, \ldots$

$\Rightarrow$ (D)

③ $f = |v|/\lambda = 44/80\,Hz = 0.55\,Hz \Rightarrow$ (B)

④ $v_g = d\omega/dk = \frac{1}{2}\sqrt{g}\,k^{-1/2} = \frac{1}{\sqrt{4}}\sqrt{\frac{g}{k}} = \sqrt{\frac{g\lambda}{8\pi}} = \sqrt{\frac{9.8}{8\pi}}\,m/s$

$\Rightarrow t = x/v_g = (300/\sqrt{9.8/8\pi})\,s \simeq 480\,s \approx 8\,min. \Rightarrow$ (B)

⑤ $v_f = \omega/k = \sqrt{g/k} = 2v_g \Rightarrow t = \frac{x}{v_f - v_g} = \frac{x}{v_g} = \frac{10}{\sqrt{9.8/8\pi}}\,s \approx 16s \Rightarrow$ (B)

⑥ $v = \sqrt{\gamma k_B T/m} = \sqrt{\frac{5}{3} \cdot 1.38 \cdot 10^{-23} \cdot 30 / 4 \cdot 1.67 \cdot 10^{-27}}\,\frac{m}{s}$

$= \sqrt{\frac{50 \cdot 1.38}{4 \cdot 1.67}} \cdot 100\,m/s \sim \sqrt{10} \cdot 100\,m/s \Rightarrow$ noe over $300\,m/s \Rightarrow$ (A)

⑦ $I(r) \sim 1/r$ for sylinderbølger $\Rightarrow \frac{I(4)}{I(20)} = 5$

$90 = 10 \log I(20) - 10 \log I_0$

$x = 10 \log I(4) - 10 \log I_0 = 10 \log 5 + 10 \log I(20) - 10 \log I_0$

$\Rightarrow x - 90 = 10 \log 5 \approx 7 \Rightarrow x = 97\,dB \Rightarrow$ (A)

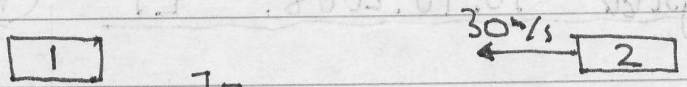
⑧ $(\pm)\vec{k}$ er forpl.retn. $\Rightarrow$ (B)

Her: $-\vec{k}$ (pga $\vec{k} \cdot \vec{r} + \omega t$ i fasen)

(9) $\lambda_1 = 2L = 7.50\,m$, $v = \sqrt{S/\mu} = \sqrt{S \cdot L / m}$

$f_1 = v/\lambda_1 = \frac{1}{2L}\sqrt{\frac{SL}{m}} = \sqrt{\frac{S}{4mL}} = \sqrt{\frac{225}{4 \cdot 0.031 \cdot 3.75}}\ Hz \approx \underline{22\,Hz} \Rightarrow$ (D)

(10)

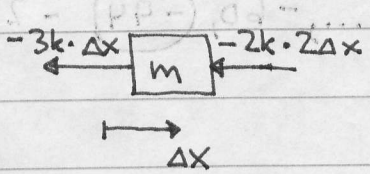


$f_1 = \frac{750}{1 - 30/340} = 823\,Hz$

$f_2 = \frac{1 + 30/340}{1} \cdot 750 = 816\,Hz$ $\Rightarrow$ (C)

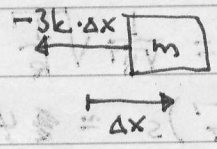
(11) $L = 3 \cdot \frac{\lambda_3}{2} \Rightarrow \frac{3}{2} \cdot \frac{v}{f_3} = 1.5 \cdot \frac{340}{4080}\ m = \underline{125\,mm} \Rightarrow$ (C)

(12) Symm. mode:



$\Rightarrow m(\ddot{\Delta x}) = -7k\Delta x \Rightarrow \omega_s^2 = 7k/m$

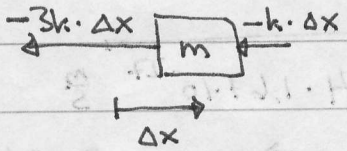
Antisymm. mode:



$\Rightarrow \omega_a^2 = 3k/m$

$\Rightarrow \omega_s/\omega_a = \sqrt{7/3} \approx 1.53 \Rightarrow$ (B)

(13)



$\Rightarrow m\ddot{\Delta x} = -4k\Delta x$

$\Rightarrow \omega^2 = 4k/m \Rightarrow$ (D)

(14) $p = -\mu\left(\int_{-a}^{0} dx + \int_{0}^{a} dx\right)\left(\frac{\partial \xi}{\partial t} \cdot \frac{\partial \xi}{\partial x}\right)$

$(-a, 0): \frac{\partial \xi}{\partial x} = +\frac{\xi_0}{a}, \quad \frac{\partial \xi}{\partial t} = -\frac{\xi_0}{a/v}$

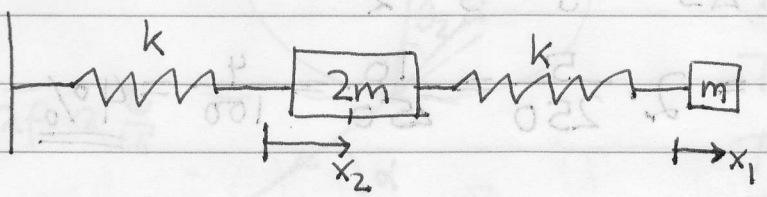
$(0, a): \frac{\partial \xi}{\partial x} = -\frac{\xi_0}{a}, \quad \frac{\partial \xi}{\partial t} = +\frac{\xi_0}{a/v}$

$\Rightarrow p = \mu \cdot \frac{\xi_0}{a} \cdot \frac{\xi_0 v}{a} \cdot (a + a) = \underline{2\mu \xi_0^2 \frac{v}{a}} \Rightarrow$ (C)

(15) $v_f = \omega/k = \sqrt{\frac{4s}{m}}\sin\left(\frac{2\pi}{d/2}\cdot\frac{d}{2}\right)\Big/\left(\frac{2\pi}{d/2}\right) \sim \sin 2\pi = \underline{0} \Rightarrow$ (A)

(16) $v_g = d\omega/dk = \frac{d}{2}\cdot\sqrt{\frac{4s}{m}}\cdot\cos 2\pi = \sqrt{\frac{sd^2}{m}} \Rightarrow$ (B)

(17)



$$2m\ddot{x}_2 = -kx_2 + k(x_1 - x_2) = -2kx_2 + kx_1$$
$$m\ddot{x}_1 = -k(x_1 - x_2) = -kx_1 + kx_2$$

Prøver $x_1 = A\cos\omega t$ og $x_2 = B\cos\omega t$!

$$\Rightarrow -2m\omega^2 B = -2kB + kA \Rightarrow -2m\omega^2 + 2k = k\cdot\frac{A}{B}$$
$$-m\omega^2 A = -kA + kB \Rightarrow (-m\omega^2 + k)\cdot\frac{A}{B} = k$$

Fra den øverste av disse to:

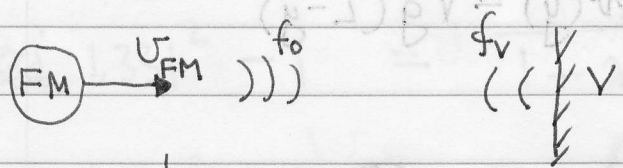
$$\frac{A}{B} = \frac{2k - 2m\omega^2}{k} \Rightarrow (k - m\omega^2)\cdot\frac{2k - 2m\omega^2}{k} = k$$

$$\Rightarrow 2k^2 - 4km\omega^2 + 2m^2\omega^4 = k^2$$

$$\Rightarrow 2m^2\omega^4 - 4km\omega^2 + k^2 = 0$$

$$\Rightarrow \omega^2 = \frac{4km \pm \sqrt{16k^2m^2 - 8k^2m^2}}{4m^2} = \frac{k}{m} \pm \frac{k}{m}\cdot\frac{\sqrt{2}}{2} = \underline{\frac{k}{m}\left(1 \pm \frac{1}{\sqrt{2}}\right)} \Rightarrow$$ (A)

(18)



$$f_v = \frac{1}{1 - v_{FM}/v}\cdot f_0$$

$$f_1 = \frac{1 + v_{FM}/v}{1}\cdot f_v = \frac{1+\alpha}{1-\alpha}\cdot f_0$$

$$\Rightarrow \frac{1+\alpha}{1-\alpha} = \frac{f_1}{f_0} = \beta \Rightarrow 1 + \alpha = \beta(1-\alpha) = \beta - \beta\alpha \Rightarrow \alpha + \beta\alpha = \beta - 1$$

$$\Rightarrow \alpha = \frac{\beta - 1}{\beta + 1} = \frac{f_1 - f_0}{f_1 + f_0} = \frac{5000}{100\,000} = \underline{5\%} \Rightarrow$$ (C)

(19) $f_s = \Delta f = 5\,\text{Hz}$

$f = v/\lambda = \frac{1}{\lambda}\sqrt{SL/m} = a\cdot\sqrt{S}$

$\Delta f = a\cdot\frac{1}{2}S^{-1/2}\cdot\Delta S = f\cdot\frac{\Delta S}{S}\cdot\frac{1}{2}$

$\Rightarrow \frac{\Delta S}{S} = 2\frac{\Delta f}{f} = 2\cdot\frac{5}{250} = \frac{10}{250} = \frac{4}{100} = \underline{\underline{4\%}} \Rightarrow$ (B)

(20) (A) $\omega(k) = a\cdot k \Rightarrow v_f = \omega/k$ ikke lineær.

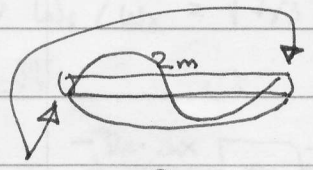
(21) (D) $\omega \sim \sqrt{k} \Rightarrow v_f \sim k^{-1/2} \sim \lambda^{1/2}$

$\Rightarrow v_f$ øker med λ (også v_g øker med λ)

(22) A er lin.pol., B er lin.pol., C er sirk.pol. $\Rightarrow$ (C)
(langs y) (langs $\hat{y}+\hat{z}$) (høyredr.?)

(D er sum av høyredr. og venstredr. sirk.pol., som blir i alt lin.pol.)

(23)



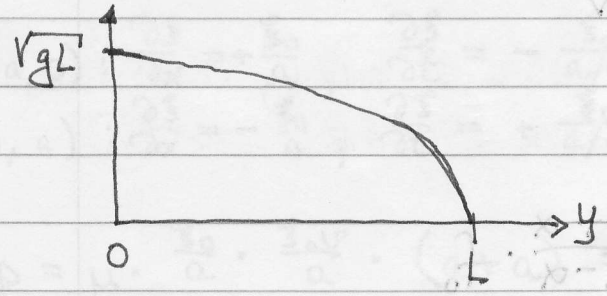
$\lambda_1 = 4\text{m}$

$\lambda_2 = 2\text{m} \Rightarrow \underline{f_2 = 340/2 = 170\,\text{Hz}}$

$\Delta p = 0$, $\xi = \xi_{max}$

$\Rightarrow$ (C)

(24) $S(y) = \mu\cdot(L-y)\cdot g \Rightarrow v(y) = \sqrt{g(L-y)}$



$\Rightarrow$ (C)

(25) Felles $S \Rightarrow v(r) = \sqrt{S/\mu}$, $v(t) = \sqrt{S/4\mu} = \frac{1}{2}v(r)$

$\Rightarrow$ kun (B) passer med dette!