F21
07.10.10

Bølgepakker og fourieranalyse (ingen bevis!)

Fourierrekke

$f(x)$ = reell, kontinuerlig, deriverbar funksjon, periodisk med periode $2L$

$$\Rightarrow \quad f(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos(nkx) + b_n \sin(nkx) \right)$$

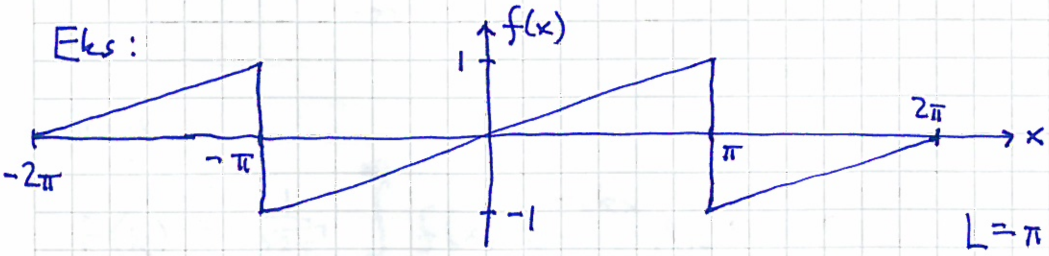
med

$$k = \frac{2\pi}{2L} = \frac{\pi}{L}$$

$$a_0 = \frac{1}{2L} \int_{-L}^{L} f(x)\,dx$$

$$\left.\begin{aligned} a_n &= \frac{1}{L} \int_{-L}^{L} f(x) \cos(nkx)\,dx \\ b_n &= \frac{1}{L} \int_{-L}^{L} f(x) \sin(nkx)\,dx \end{aligned}\right\} \quad n = 1, 2, 3, \ldots$$

Eks:



$L = \pi \Rightarrow k = 1$

$$\Rightarrow \quad a_0 = 0; \qquad a_1 = a_2 = \ldots = a_n = 0$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} \frac{x}{\pi} \cdot \sin(nx)\,dx = \ldots = \frac{2}{\pi n} (-1)^{n+1} \qquad (n = 1, 2, 3, \ldots)$$

$$\Rightarrow \quad f(x) = \frac{2}{\pi} \left\{ \sin x - \frac{1}{2} \sin 2x + \frac{1}{3} \sin 3x \ldots\ldots \right\}$$

Fourier integral

$f(x)$ = reell, kontinuerlig, deriverbar funksjon

$$\Rightarrow \quad f(x) = \int_0^\infty \left[g(k)\cos(kx) + h(k)\sin(kx) \right] dx$$

med

$$g(k) = \frac{1}{\pi} \int_{-\infty}^{\infty} f(x)\cos(kx)\,dx$$

$$h(k) = \frac{1}{\pi} \int_{-\infty}^{\infty} f(x)\sin(kx)\,dx$$

Fourier-transform

Kompleks form av Fourierintegral.

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(k)\,e^{ikx}\,dk$$

med

$$F(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)\,e^{-ikx}\,dx$$

Med bølger:

Tilsvarende, men $kx \rightarrow kx - \omega t$, med $\omega = \omega(k)$

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$$f(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(\omega)\,e^{i\omega t}\,d\omega \,; \qquad F(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t)\,e^{-i\omega t}\,dt$$