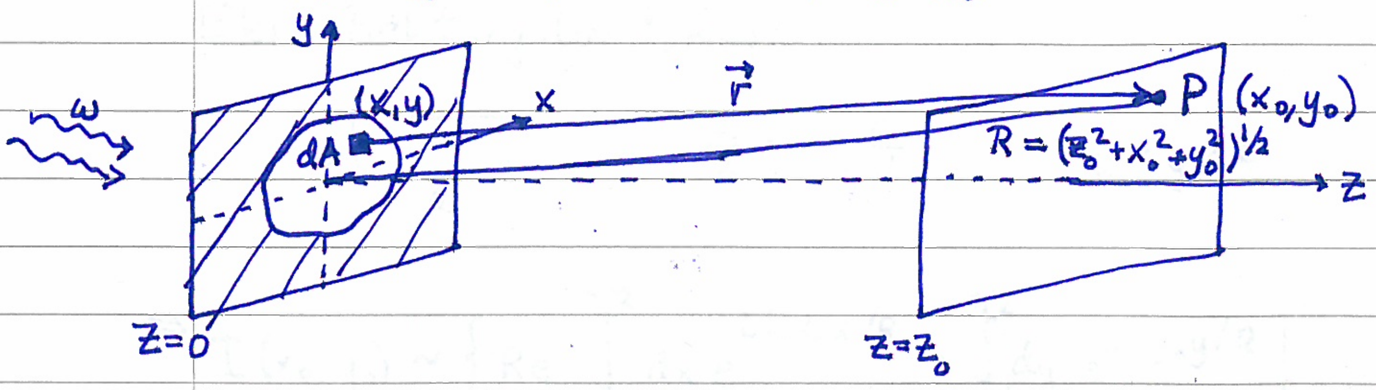
Diffraksjon fra liten åpning



felt i P fra punktkilde på flateelement dA:

[Huygens!]

$$dE = \frac{C}{r}\cos(kr - \omega t)\cdot dA = \mathrm{Re}\left\{\frac{C}{r}\, e^{i(\omega t - kr)}\, dA\right\}$$

$$\Rightarrow E = \int dE = \mathrm{Re}\left\{\int_{\text{åpning}} \frac{C}{r}\, e^{i(\omega t - kr)}\, dA\right\}$$

= felt i P fra åpningen

$$\frac{C}{r} \approx \frac{C}{R} \quad \text{uavh. av } (x,y)$$

Tilnærming for r i fasen:

$$r = \sqrt{z_0^2 + (x_0 - x)^2 + (y_0 - y)^2} = \sqrt{z_0^2 + x_0^2 + y_0^2 - 2x_0 x - 2y_0 y + x^2 + y^2}$$

$$\approx R\left(1 - \frac{x_0 x + y_0 y}{R^2}\right)$$

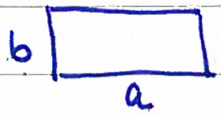
der vi antar $x \ll x_0$; $y \ll y_0$

Dermed:

$$E = \mathrm{Re}\left\{\frac{C}{R}\, e^{i(\omega t - kR)} \int_{\text{åpning}} dA\, e^{ik(x_0 x + y_0 y)/R}\right\}$$

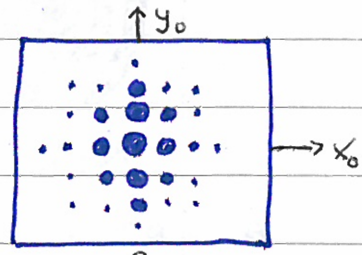
$$I \sim \langle E^2 \rangle \sim \left\{\mathrm{Re} \int_{\text{åpning}} dA\, e^{ik(x_0 x + y_0 y)/R}\right\}^2$$

Eks: Rektangulær åpning



$-\frac{a}{2} < x < \frac{a}{2} \qquad -\frac{b}{2} < y < \frac{b}{2}$

$$\Rightarrow I(x_0, y_0) \sim \left\{ \mathrm{Re} \int_{-a/2}^{a/2} dx\, e^{ikx_0 x/R} \cdot \int_{-b/2}^{b/2} dy\, e^{iky_0 y/R} \right\}^2$$



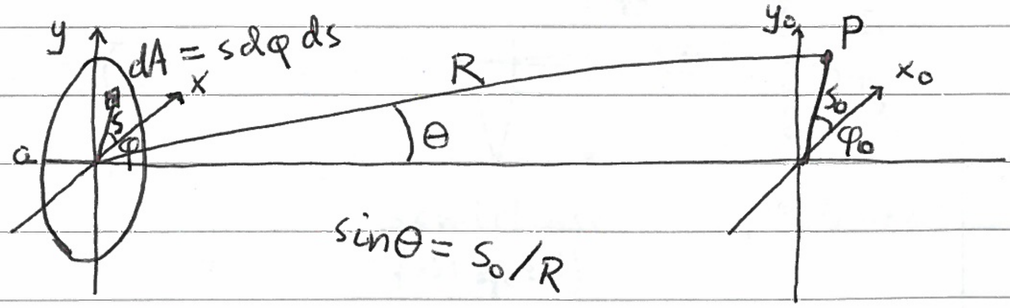
$$\sim \left(\frac{\sin\alpha}{\alpha}\right)^2 \cdot \left(\frac{\sin\beta}{\beta}\right)^2$$

med $\alpha = kx_0 a/2R$, $\beta = ky_0 b/2R$

[Kommentar: ser at $I \sim \{F(kx_0/R) \cdot G(ky_0/R)\}^2$ der F, G er Fouriertransformasjoner: $F(k) = \int_{-\infty}^{\infty} f(x) e^{ikx} dx$ med $f(x) = \begin{cases} 1 & |x| < \frac{a}{2} \\ 0 & \text{ellers} \end{cases}$]

Eks: Sirkulær åpning

Symmetri $\Rightarrow I(x_0, y_0) \rightarrow I(s_0)$; $\quad s_0 = \sqrt{x_0^2 + y_0^2}$



$x = s\cos\varphi$, $y = s\sin\varphi$, $x_0 = s_0\cos\varphi_0$, $y_0 = s_0 \sin\varphi_0$

$$\Rightarrow x_0 x + y_0 y = s s_0 \cos(\varphi - \varphi_0)$$

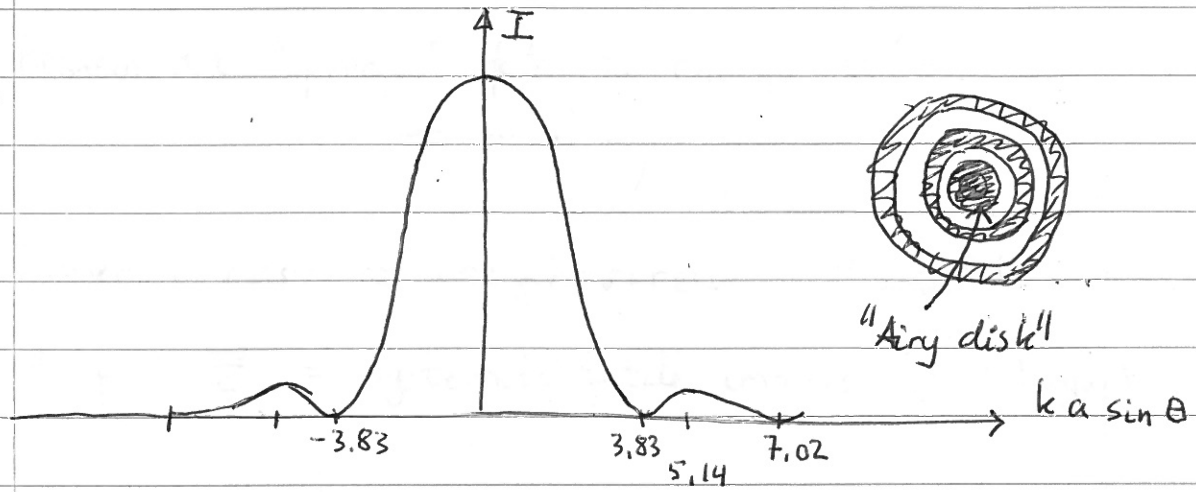
(Integral over s kan gjøres med delvis int.!)

$$\Rightarrow I \sim \left\{ \mathrm{Re} \int_0^a ds \int_0^{2\pi} d\varphi \; s \; e^{ikss_0\cos(\varphi-\varphi_0)/R} \right\}^2 \sim \dots \sim$$

$$\sim \left\{ J_1(u)/u \right\}^2 ; \quad u = ka\sin\theta = 2\pi a \sin(\theta)/\lambda$$

$$J_1(u) = \frac{1}{2\pi i}\int_0^{2\pi} e^{i(\varphi + u\cos\varphi)}\, d\varphi$$

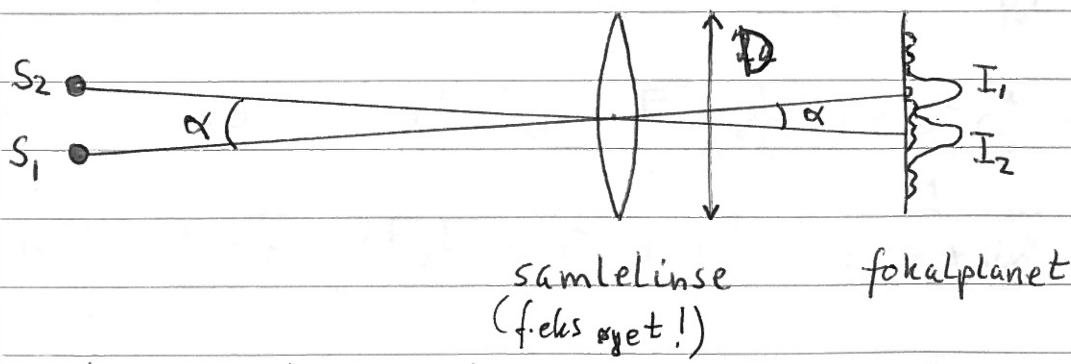
$= 1.$ ordens Besselfunksjon av 1. slag



1. nullpunkt: $\sin\Theta \approx \Theta \approx 3.83\,\lambda/2\pi a \approx 1.22\,\lambda/2a$

Oppløsningsevne:
Ser to (sirkulære) kilder S_1 og S_2 adskilt bare hvis I_1 og I_2 ikke overlapper for mye:

(ikke punktkilder)



$\Rightarrow$ oppløsning begrenset av <u>diffraksjon</u> hvis $\alpha \lesssim 1.22\lambda/D$

stor D $\Rightarrow$ god oppløsning $\Rightarrow$ bra med store teleskoper

$\lambda_{Blå} < \lambda_{Rød}$ $\Rightarrow$ bedre oppløsning for blått enn for rødt lys

Hit 7.11.08