TFY4215 Innføring i kvantefysikk. Institutt for fysikk, NTNU. Våren 2018.
Compulsory exercise. Deadline: Sunday April 8, kl 23.59.

To be delivered on blackboard, individually or in groups of two or three.
Alternative 1:
Produce a tidy report in LaTeX, including figures, in pdf format. In addition, hand in a working code, normally in python or matlab.
Alternative 2:
Make an IPython (Jupyter) notebook, where text, figures and python code are integrated into a single document. Eilif Sommer Øyre will give an introduction to IPython notebook on Wednesday March 7 (R5 kl 12.15). See also the webpage numfys.net.

Wave packets and Heisenberg's uncertainty relation

1. Let a gaussian wave packet $\Psi(x,0)$ represent a free electron ($V = 0$) with mean starting position $\langle x\rangle(0) = x_0$, average momentum $\langle p\rangle(0) = p_0$ and uncertainty (standard deviation) $\Delta x(0) = \sigma$. Compute $\Delta x(t)$ numercally and compare with the analytic expression for $\Delta x(t)$ by plotting both in a single figure.

2. Next, let the potential be harmonic, $V(x) \sim x^2$. Use initial states $\Psi(x,0)$ as in the previous questions, and study their propagation in space and time, $\Psi(x,t)$, as well as the time evolution of the uncertainty in position and momentum, i.e., $\Delta x(t)$ and $\Delta p(t)$. Check in particular that Heisenberg's uncertainty relation is satisfied. Plot the uncertainty product as a function of time t. Use at least two different initial states, that produce qualitatively different behavior for $\Delta x(t)$. Comment upon expected and unexpected behavior.

Some hints:

- Add sufficiently large regions with constant potential on each side of the harmonic potential. The initial state $\Psi(x,0)$ should be mostly localized in between the two classical turning points (corresponding to the average momentum p_0, i.e., the energy $E = p_0^2/2m$).
- Use the exercises (question nr 7 and 11) as guidance.
- See lecture notes and exercises (question 15) for how to evaluate the coefficients c_n in

$$\Psi(x,t) = \sum_{n=0}^{N-1} c_n \psi_n(x)\, \exp(-iE_n t/\hbar).$$

- See lecture notes and available python programs for a numerical solution of TUSL.
- You are encouraged to make your programs so they include animations of the probability densities $|\Psi(x,t)|^2$.