TFY4215 Innføring i kvantefysikk Midterm 1. mars 2019 Formelvedlegg

- Partikkel i boks

$$V(x) = 0 \quad \text{for} \quad 0 < x < L, \quad V(x) = \infty \ \text{ ellers}$$

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\frac{n\pi x}{L} \quad , \quad E_n = \frac{n^2\pi^2\hbar^2}{2mL^2} \quad , \quad n = 1, 2, 3, \ldots$$

- Endimensjonal harmonisk oscillator

$$\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \tfrac{1}{2}m\omega^2 x^2\right)\psi_n(x) = \hbar\omega(n+\tfrac{1}{2})\psi_n(x); \qquad \langle\, \psi_n, \psi_k \,\rangle = \delta_{nk};$$

$$\psi_n(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \frac{1}{\sqrt{2^n\, n!}}\, e^{-y^2/2} H_n(y), \qquad y = \frac{x}{\sqrt{\hbar/m\omega}};$$

$$H_0(y) = 1, \quad H_1(y) = 2y, \quad H_2(y) = 4y^2 - 2, \quad H_3(y) = 8y^3 - 12y, \quad \cdots;$$

$$\widehat{\mathcal{P}}\psi_n(x) \equiv \psi_n(-x) = (-1)^n \psi_n(x).$$

- Noen konstanter

$$a_0 = \frac{4\pi\epsilon_0\hbar^2}{m_e e^2} \approx 0.529 \cdot 10^{-10} \text{ m} \qquad \text{(Bohr-radien);}$$

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \approx \frac{1}{137.0360} \qquad \text{(finstrukturkonstanten);}$$

$$\tfrac{1}{2}\alpha^2 m_e c^2 = \frac{\hbar^2}{2m_e a_0^2} \approx 13.6 \text{ eV} \qquad \text{(Rydberg-energien).}$$

$$m_e \simeq 9.11 \cdot 10^{-31} \text{ kg} \qquad \hbar = h/2\pi \simeq 1.05 \cdot 10^{-34} \text{ Js} \qquad e \simeq 1.60 \cdot 10^{-19} \text{ C} \qquad u \simeq 1.66 \cdot 10^{-27} \text{ kg}$$

$$m_p \simeq m_n \simeq 1.67 \cdot 10^{-27} \text{ kg} \qquad k_B \simeq 1.38 \cdot 10^{-23} \text{ J/K} \qquad c \simeq 3.00 \cdot 10^8 \text{ m/s} \qquad 1 \text{ Å} = 0.1 \text{ nm}$$

$$1/4\pi\epsilon_0 \simeq 9 \cdot 10^9 \text{ Nm}^2/\text{C}^2 \qquad m_e c^2 \simeq 511 \text{ keV} \qquad m_p c^2 \simeq m_n c^2 \simeq 939 \text{ MeV}$$

- Noen formler

$$\sin a = (e^{ia} - e^{-ia})/2i \ , \qquad \cos a = (e^{ia} + e^{-ia})/2;$$

$$\tan y = \frac{1}{\cot y} = \tan(y + n\pi), \quad n = 0, \pm 1, \cdots;$$

$$\sinh y = \tfrac{1}{2}(e^y - e^{-y}); \qquad \cosh y = \tfrac{1}{2}(e^y + e^{-y}); \qquad \tanh y = \frac{1}{\coth y} = \frac{\sinh y}{\cosh y};$$

$$\cosh^2 y - \sinh^2 y = 1; \qquad \frac{d}{dy}\sinh y = \cosh y; \qquad \frac{d}{dy}\cosh y = \sinh y.$$

$$|y| \ll 1 \Rightarrow \exp(y) \simeq 1 + y$$

$$|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 + 2\text{Re}(z_1^* z_2)$$

- de Broglie:

$$\lambda = h/p \ , \ \ \nu = E/h$$

- Midlere translasjonsenergi pr partikkel i ideell gass (i 3 dimensjoner): $3k_BT/2$

- Schrödingerligningen:
$$i\hbar\frac{\partial\Psi}{\partial t} = \widehat{H}\Psi$$
- Tidsuavhengig Schrödingerligning:
$$\widehat{H}\psi = E\psi$$
- Impulsoperator:
$$\widehat{p}_x = \frac{\hbar}{i}\frac{\partial}{\partial x} \quad , \quad \widehat{\boldsymbol{p}} = \frac{\hbar}{i}\nabla \quad , \quad f(p) \to f(\widehat{p})$$
- Kinetisk energi:
$$K = \frac{p^2}{2m}$$
- Dreieimpuls:
$$\boldsymbol{L} = \boldsymbol{r} \times \boldsymbol{p}$$
- Heisenbergs uskarphetsprinsipp:
$$\Delta x \Delta p \geq \hbar/2$$
$$\Delta A \Delta B \geq \frac{1}{2}\left|\langle i[\widehat{A},\widehat{B}]\rangle\right|$$
- Kommutator:
$$[\widehat{A},\widehat{B}] = \widehat{A}\widehat{B} - \widehat{B}\widehat{A}$$
- Stasjonær tilstand:
$$\Psi(x,t) = \psi(x)e^{-iEt/\hbar}$$
- Forventningsverdier:
$$\begin{aligned} \langle x\rangle &= \int \Psi^* x \Psi dx \\ \langle p\rangle &= \int \Psi^* \frac{\hbar}{i}\frac{\partial}{\partial x}\Psi dx \\ \langle F\rangle &= \int \Psi^* \widehat{F}\Psi d\tau \end{aligned}$$
- Bølgepakke:
$$\Psi(x,t) = \sum_j c_j \psi_j(x) e^{-iE_j t/\hbar}$$
$$c_j = \int \psi_j^*(x)\Psi(x,0)dx$$
- Grensebetingelser:
$\psi(x)$ kontinuerlig overalt, $d\psi/dx$ diskontinuerlig ved ∞ sprang i $V(x)$
- Sannsynlighetsstrøm:
$$j = \mathrm{Re}\left[\Psi^*\left(\frac{\hbar}{mi}\frac{\partial}{\partial x}\right)\Psi\right]$$
- Usikkerhet (standardavvik):
$$\Delta x = \sqrt{\langle x^2\rangle - \langle x\rangle^2} \quad , \quad \Delta p = \sqrt{\langle p^2\rangle - \langle p\rangle^2}$$
- Ehrenfests teorem:
$$\frac{d}{dt}\langle \boldsymbol{r}\rangle = \frac{\langle \boldsymbol{p}\rangle}{m} \quad , \quad \frac{d}{dt}\langle \boldsymbol{p}\rangle = -\langle \nabla V\rangle$$
- Relativistisk energi (K er kinetisk energi):
$$E^2 = p^2c^2 + m^2c^4 \quad ; \quad E = K + mc^2$$
- Prefikser:
f = 10^{-15}, p = 10^{-12}, n = 10^{-9}, $\mu = 10^{-6}$, m = 10^{-3}, k = 10^3, M = 10^6, G = 10^9, T = 10^{12}
- Et par potensielt nyttige tallverdier:
$\hbar^2/2m_e = 0.0378$ eV nm^2
$hc = 1237$ eV nm