

FORMLER TFY4215 Innføring i kvantefysikk Eksamen 12. august 2021

- Partikkel i boks

$$V(x) = 0 \quad \text{for} \quad 0 < x < L, \quad V(x) = \infty \quad \text{ellers}$$

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}, \quad E_n = \frac{n^2 \pi^2 \hbar^2}{2mL^2}, \quad n = 1, 2, 3, \dots$$

- Endimensjonal harmonisk oscillator

$$\left(-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + \frac{1}{2} m \omega^2 x^2 \right) \psi_n(x) = \hbar \omega \left(n + \frac{1}{2} \right) \psi_n(x); \quad \langle \psi_n, \psi_k \rangle = \delta_{nk};$$

$$\psi_n(x) = \left(\frac{m\omega}{\pi \hbar} \right)^{1/4} \frac{1}{\sqrt{2^n n!}} e^{-y^2/2} H_n(y), \quad y = \frac{x}{\sqrt{\hbar/m\omega}};$$

$$H_0(y) = 1, \quad H_1(y) = 2y, \quad H_2(y) = 4y^2 - 2, \quad H_3(y) = 8y^3 - 12y, \quad \dots;$$

$$\hat{\mathcal{P}}\psi_n(x) \equiv \psi_n(-x) = (-1)^n \psi_n(x).$$

- Kulekoordinater: $z = r \cos \theta$, $x = r \sin \theta \cos \phi$, $y = r \sin \theta \sin \phi$
- Laplace-operatoren og dreieimpulsoperatorer i kulekoordinater

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} - \frac{\hat{\mathbf{L}}^2}{\hbar^2 r^2};$$

$$\hat{\mathbf{L}}^2 = -\hbar^2 \left(\frac{\partial^2}{\partial \theta^2} + \cot \theta \frac{\partial}{\partial \theta} + \frac{1}{\sin^2 \theta} \frac{\partial^2}{\partial \phi^2} \right), \quad \hat{L}_z = \frac{\hbar}{i} \frac{\partial}{\partial \phi};$$

$$\hat{L}_x = \frac{\hbar}{i} \left(-\sin \phi \frac{\partial}{\partial \theta} - \cot \theta \cos \phi \frac{\partial}{\partial \phi} \right), \quad \hat{L}_y = \frac{\hbar}{i} \left(\cos \phi \frac{\partial}{\partial \theta} - \cot \theta \sin \phi \frac{\partial}{\partial \phi} \right);$$

$$[\hat{\mathbf{L}}^2, \hat{L}_z] = 0, \quad [\hat{L}_x, \hat{L}_y] = i\hbar \hat{L}_z, \quad \text{osv.}$$

- Vinkelfunksjoner

$$\left\{ \begin{array}{c} \hat{\mathbf{L}}^2 \\ \hat{L}_z \end{array} \right\} Y_{lm} = \left\{ \begin{array}{c} \hbar^2 l(l+1) \\ \hbar m \end{array} \right\} Y_{lm}, \quad l = 0, 1, 2, \dots; \quad m = 0, \pm 1, \pm 2, \dots, \pm l$$

$$\int_0^{2\pi} d\phi \int_{-1}^1 d(\cos \theta) Y_{l'm'}^* Y_{lm} = \delta_{l'l} \delta_{m'm};$$

$$Y_{00} = \frac{1}{\sqrt{4\pi}}, \quad Y_{10} = \sqrt{\frac{3}{4\pi}} \cos \theta = \sqrt{\frac{3}{4\pi}} \frac{z}{r} \equiv Y_{p_z}, \quad Y_{1\pm 1} = \mp \sqrt{\frac{3}{8\pi}} \sin \theta e^{\pm i\phi};$$

$$Y_{p_x} = \sqrt{\frac{3}{4\pi}} \frac{x}{r} = \frac{1}{\sqrt{2}} (Y_{1,-1} - Y_{11}), \quad Y_{p_y} = \sqrt{\frac{3}{4\pi}} \frac{y}{r} = \frac{i}{\sqrt{2}} (Y_{11} + Y_{1,-1});$$

$$Y_{20} = \sqrt{\frac{5}{16\pi}} (3 \cos^2 \theta - 1); \quad Y_{2,\pm 1} = \mp \sqrt{\frac{15}{8\pi}} \sin \theta \cos \theta e^{\pm i\phi}; \quad Y_{2,\pm 2} = \sqrt{\frac{15}{32\pi}} \sin^2 \theta e^{\pm 2i\phi}.$$

$$\hat{\mathcal{P}} Y_{lm} = (-1)^l Y_{lm}.$$

- Energieeigenfunksjoner og radiallyigning, kulesymmetrisk potensial $V(r)$

$$\psi(r, \theta, \phi) = \frac{u(r)}{r} Y_{lm}(\theta, \phi);$$

$$\left[-\frac{\hbar^2}{2m} \frac{d^2}{dr^2} + V_{\text{eff}}^l(r) \right] u(r) = E u(r), \quad V_{\text{eff}}^l(r) \equiv V(r) + \frac{\hbar^2 l(l+1)}{2mr^2}, \quad u(0) = 0.$$

- Energieeigenverdier og -eigenfunksjoner, hydrogenatomet, $V(r) = -e^2/(4\pi\epsilon_0 r)$

$$E_n = \frac{E_1}{n^2} \equiv \frac{E_1}{(l+1+n_r)^2}, \quad E_1 = -\frac{1}{2}\alpha^2 m_e c^2;$$

$$\psi_{nlm} = R_{nl}(r) Y_{lm}(\theta, \phi);$$

$$R_{10} = \frac{2}{a_0^{3/2}} e^{-r/a_0}; \quad R_{20} = \frac{1}{\sqrt{2} a_0^{3/2}} \left(1 - \frac{r}{2a_0} \right) e^{-r/2a_0}; \quad R_{21} = \frac{1}{2\sqrt{6} a_0^{3/2}} \frac{r}{a_0} e^{-r/2a_0}.$$

- de Broglie:

$$\lambda = h/p, \quad \nu = E/h$$

- Midlere translasjonsenergi pr partikkel i ideell gass (i 3 dimensjoner): $3k_B T/2$
- Schrödingerligningen:

$$i\hbar \frac{\partial \Psi}{\partial t} = \hat{H} \Psi$$

- Tidsuavhengig Schrödingerligning:

$$\hat{H} \psi = E \psi$$

- Impulsoperator:

$$\hat{p}_x = \frac{\hbar}{i} \frac{\partial}{\partial x}, \quad \hat{\mathbf{p}} = \frac{\hbar}{i} \nabla, \quad f(p) \rightarrow f(\hat{p})$$

- Kinetisk energi:

$$K = \frac{p^2}{2m}$$

- Dreieimpuls:

$$\mathbf{L} = \mathbf{r} \times \mathbf{p}$$

- Heisenbergs uskarphetsprinsipp:

$$\Delta x \Delta p \geq \hbar/2$$

$$\Delta A \Delta B \geq \frac{1}{2} \left| \langle i[\hat{A}, \hat{B}] \rangle \right|$$

- Kommutator:

$$[\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}$$

- Stasjonær tilstand:

$$\Psi(x, t) = \psi(x) e^{-iEt/\hbar}$$

- Forventningsverdier:

$$\begin{aligned}\langle x \rangle &= \int \Psi^* x \Psi dx \\ \langle p \rangle &= \int \Psi^* \frac{\hbar}{i} \frac{\partial}{\partial x} \Psi dx \\ \langle F \rangle &= \int \Psi^* \hat{F} \Psi d\tau\end{aligned}$$

- Bølgepakke:

$$\begin{aligned}\Psi(x, t) &= \sum_j c_j \psi_j(x) e^{-iE_j t/\hbar} \\ c_j &= \int \psi_j^*(x) \Psi(x, 0) dx\end{aligned}$$

- Grensebetingelser:
 $\psi(x)$ kontinuerlig overalt, $d\psi/dx$ diskontinuerlig ved ∞ sprang i $V(x)$
- Sannsynlighetsstrøm:

$$j = \text{Re} \left[\Psi^* \left(\frac{\hbar}{mi} \frac{\partial}{\partial x} \right) \Psi \right]$$

- Usikkerhet (standardavvik):

$$\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} \quad , \quad \Delta p = \sqrt{\langle p^2 \rangle - \langle p \rangle^2}$$

- Ehrenfests teorem:

$$\frac{d}{dt} \langle \mathbf{r} \rangle = \frac{\langle \mathbf{p} \rangle}{m} \quad , \quad \frac{d}{dt} \langle \mathbf{p} \rangle = -\langle \nabla V \rangle$$

- Relativistisk energi (K er kinetisk energi):

$$E^2 = p^2 c^2 + m^2 c^4 \quad ; \quad E = K + mc^2$$

- Prefikser:

$$\text{f} = 10^{-15}, \text{p} = 10^{-12}, \text{n} = 10^{-9}, \mu = 10^{-6}, \text{m} = 10^{-3}, \text{k} = 10^3, \text{M} = 10^6, \text{G} = 10^9, \text{T} = 10^{12}$$

- Noen konstanter

$$a_0 = \frac{4\pi\epsilon_0\hbar^2}{m_e e^2} \approx 0.529 \cdot 10^{-10} \text{ m} \quad (\text{Bohr-radialen});$$

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \approx \frac{1}{137.0360} \quad (\text{finstrukturkonstanten});$$

$$\frac{1}{2}\alpha^2 m_e c^2 = \frac{\hbar^2}{2m_e a_0^2} \approx 13.6 \text{ eV} \quad (\text{Rydberg-energien}).$$

$$m_e \simeq 9.11 \cdot 10^{-31} \text{ kg} \quad \hbar = h/2\pi \simeq 1.05 \cdot 10^{-34} \text{ Js} \quad e \simeq 1.60 \cdot 10^{-19} \text{ C} \quad u \simeq 1.66 \cdot 10^{-27} \text{ kg}$$

$$m_p \simeq m_n \simeq 1.67 \cdot 10^{-27} \text{ kg} \quad k_B \simeq 1.38 \cdot 10^{-23} \text{ J/K} \quad c \simeq 3.00 \cdot 10^8 \text{ m/s} \quad 1 \text{ \AA} = 0.1 \text{ nm}$$

$$1/4\pi\epsilon_0 \simeq 9 \cdot 10^9 \text{ Nm}^2/\text{C}^2 \quad m_e c^2 \simeq 511 \text{ keV} \quad m_p c^2 \simeq m_n c^2 \simeq 939 \text{ MeV}$$

- Et par potensielt nyttige tallverdier: $\hbar^2/2m_e = 0.0378 \text{ eV nm}^2$ $hc = 1237 \text{ eV nm}$

- Noen formler

$$\sin a = (e^{ia} - e^{-ia})/2i, \quad \cos a = (e^{ia} + e^{-ia})/2;$$

$$\tan y = \frac{1}{\cot y} = \tan(y + n\pi), \quad n = 0, \pm 1, \dots;$$

$$\sinh y = \frac{1}{2}(e^y - e^{-y}); \quad \cosh y = \frac{1}{2}(e^y + e^{-y}); \quad \tanh y = \frac{1}{\coth y} = \frac{\sinh y}{\cosh y};$$

$$\cosh^2 y - \sinh^2 y = 1; \quad \frac{d}{dy} \sinh y = \cosh y; \quad \frac{d}{dy} \cosh y = \sinh y.$$

$$|y| \ll 1 \Rightarrow \exp(y) \simeq 1 + y$$

$$|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 + 2\operatorname{Re}(z_1^* z_2)$$

- Utvalgsregler for strålingsoverganger: $\Delta l = \pm 1$, $\Delta m = 0$ eller ± 1
- Spinn-1/2-partikler:

$$\text{Egentilstander : } \chi_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \chi_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\text{Operatorer : } \hat{S}_x = \frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \hat{S}_y = \frac{\hbar}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \hat{S}_z = \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \hat{S}^2 = \frac{3\hbar^2}{4} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\text{Normering : } \chi^\dagger \chi = (a^* \ b^*) \begin{pmatrix} a \\ b \end{pmatrix} = |a|^2 + |b|^2 = 1$$

$$\text{Forventningsverdi : } \langle S_j \rangle = \chi^\dagger \hat{S}_j \chi$$

$$\text{Standardavvik : } \Delta S_j = \sqrt{\langle S_j^2 \rangle - \langle S_j \rangle^2}$$