
Problem 1. *Use the fact that $L_j = \mathbf{x} \cdot \mathbf{p} = \epsilon_{jlm} x_l p_m$, and that you can rewrite ϵ_{jlm} indices. Recall as well that $[x_i, p_j] = \delta_{ij}$*

$$[p_i, L_j] = [p_i, \epsilon_{jlm} x_l p_m] = [p_i, x_l]\epsilon_{jlm} p_m = -\epsilon_{jim} p_m = \epsilon_{ijk} p_k \tag{1}$$

$$[x_i, L_j] = [x_i, \epsilon_{jlm} x_l p_m] = [x_i, p_m]\epsilon_{jlm} p_m = -\epsilon_{jli} x_l = \epsilon_{ijk} x_k \tag{2}$$

Problem 2. *Lorentz transformation for velocity components*

$$x' = \gamma[x - \beta ct]; \quad y' = y, z' = z, \quad z' = \gamma\left(t - \frac{\beta}{c}x\right) \quad \overset{\text{diff}}{\to}$$

$$dx' = \gamma[dx - \beta c dt], \quad dy' = dy, \quad dz' = dz, \quad dt' = \gamma\left(dt - \frac{\beta}{c}dx\right)$$

$$v'_x = \frac{dx'}{dt'} = \frac{\gamma(dx - \beta c dt)}{\gamma(dt - \beta/c dx)} = \frac{dx - \beta c dt}{dt - \beta/c dx} \cdot \frac{\frac{1}{dt}}{\frac{1}{dt}}$$

$$\Rightarrow \frac{\frac{dx}{dt} - \beta c \frac{dt}{dt}}{\frac{dt}{dt} - \frac{\beta}{c}\frac{dx}{dt}} = \frac{v_x - \beta c}{1 - \frac{\beta}{c}v_x} = \frac{v_x - \beta c}{1 - \beta^2}, \quad \frac{1}{\gamma} = \sqrt{1 - \beta^2}$$

$$v'_y = \frac{dy'}{dt} = \frac{dy}{\gamma(t - \beta/c dx)} = \frac{v_y}{\gamma\left(-1 - \beta^2\right)}, \quad \frac{1}{\gamma} = \sqrt{1 - \beta^2}$$

$$\Rightarrow v'_y = \frac{v_y}{\sqrt{1 - \beta^2}}, \quad v'_z = \frac{v_z}{\sqrt{1 - \beta^2}}$$

$$a'_x = \frac{dv'_x}{dt'} \Longrightarrow a'_x = \frac{dv_x}{\left(1 - \beta^2\right)\gamma\left(dt - \frac{\beta}{c}dx\right)} = \frac{a_x}{\left(1 - \beta^2\right)\sqrt{1 - \beta^2}}$$

$$= \frac{a_x}{\left(1 - \beta^2\right)^{3/2}}, \quad a'_y = \frac{dv'_y}{dt'} = \frac{dv_y/dt}{\sqrt{1 - \beta^2}\gamma^2\left(1 - \beta^2\right)}$$

$$\Rightarrow a'_y = \frac{a_y}{1 - \beta^2}, \quad a'_z = \frac{a_z}{1 - \beta^2}$$

Problem 3. *Problem 3.*

Let dN be the number of wavelengths emitted from the ^{57}Co in the time interval dt' in the system S' in which the atom is instantaneous at rest. $dt' = d\tau$ is the eigentime of S'. The frequency as measured in S' is

$$f_0 = \frac{dN}{dt'}$$

If t is the time as measured by the observer at the center, we have $dt = \gamma d\tau$. The frequency in the center is then

$$f = \frac{dN}{dt} = \frac{1}{\gamma}\frac{dN}{d\tau} = \sqrt{1-\beta^2} f_0$$

The transversal Doppler effect.