TFY4345 Classical Mechanics. Department of Physics, NTNU.

SOLUTION ASSIGNMENT 11

Question 1

Since the total 3-momentum is zero before and after the collision, all the kinetic energy can be converted into rest energy, i.e., mass. Thus, each colliding particle must have a kinetic energy of at least $T = mc^2 = 938$ MeV.
An analysis similar to the case with the neutron initially at rest, but now with $\boldsymbol{p}_n = -\boldsymbol{p}_p$ and $E_n = E_p = mc^2 + T$, gives the same result.

Question 2

$iE_2'/c = F_{42}' = L_{4\alpha}L_{2\beta}F_{\alpha\beta} = L_{4i}L_{22}F_{i2} = -i\beta\gamma\cdot(-B_1) + \gamma iE_2/c \;\;\Rightarrow\;\; E_2' = \gamma(E_2 + vB_1)$

$iE_3'/c = F_{43}' = L_{4\alpha}L_{3\beta}F_{\alpha\beta} = -i\beta\gamma L_{3\beta}F_{3\beta} + \gamma L_{3\beta}F_{4\beta} = -i\beta\gamma i\beta\gamma\cdot(-iE_3/c) + \gamma^2\cdot(iE_3/c) = (i/c)\gamma^2 E_3(1-\beta^2) = iE_3/c \;\;\Rightarrow\;\; E_3' = E_3$

$B_2' = F_{31}' = L_{3\alpha}L_{1\beta}F_{\alpha\beta} = L_{3\alpha}L_{11}F_{\alpha 1} = \gamma B_2 + i\beta\gamma\cdot(iE_1/c) \;\;\Rightarrow\;\; B_2' = \gamma(B_2 - \beta E_1/c)$

$B_3' = F_{12}' = L_{1\alpha}L_{2\beta}F_{\alpha\beta} = L_{1\alpha}F_{\alpha 2} = B_3$

Question 3

Energy conservation:

$$m_\pi c^2 = m_\mu c^2 + T + pc,$$

i.e.,

$$pc = m_\pi c^2 - m_\mu c^2 - T.$$

Here, pc is the energy of the neutrino. For the energy of the muon, we have both $E_\mu = m_\mu c^2 + T$ and $E_\mu^2 = (m_\mu c^2)^2 + (pc)^2$, since the two created particles have equal absolute value p of the 3-momentum. Thus,

$$(m_\mu c^2 + T)^2 = (m_\mu c^2)^2 + (m_\pi c^2 - m_\mu c^2 - T)^2.$$

Some terms cancel, and we are left with

$$T = \frac{(m_\pi - m_\mu)^2 c^2}{2m_\pi}.$$

Question 4

a) ..holonomic. (B)

b) $\varepsilon_{333} = 0$ (E)

c) Js, i.e., the same unit as for angular momentum (A)

d) ..cyclic (E)

e) $a_x = -(\partial V/\partial x)/m = -2\alpha xy^2/m = 0$ on the y axis (where $x = 0$). $a_y = -(\partial V/\partial y)/m = -2\alpha x^2 y/m = 0$ on the y axis. (A)