TFY4345 Classical Mechanics. Department of Physics, NTNU.

ASSIGNMENT 10

Question 1

The inertial system S' is moving with velocity v relative to system S along the x_3 axis. The inertial system S'' is moving with velocity v' relative to S' in the same direction. Show that S'' is moving with velocity

$$v'' = \frac{v + v'}{1 + vv'/c^2}$$

relative to S.
Hint: Use the transformation equations $\boldsymbol{x}' = \boldsymbol{L}\boldsymbol{x}$ and $\boldsymbol{x}'' = \boldsymbol{L}'\boldsymbol{x}' = \boldsymbol{L}'\boldsymbol{L}\boldsymbol{x} = \boldsymbol{L}''\boldsymbol{x}$ and compare the matrix elements of $\boldsymbol{L}''$ with those of the matrix product $\boldsymbol{L}'\boldsymbol{L}$.

Question 2 (Exam 2016)

A fluorescent tube lamp is stationary in a reference frame S and lies along the z axis. The tube lights up simultaneously (in S) along its entire length L_0 at time t. The tube has one end at $z = 0$ and the other at $z = L_0$. An observer at rest in the reference frame S' is moving with velocity v relative to S along the z axis.

a) Consider two events in S, the lighting up of the tube in the two positions z and $z + \Delta z$, both at time t. Use the Lorentz transformation to find the spacetime coordinates of these two events in S', i.e., z' at time t' and $z' + \Delta z'$ at time $t' + \Delta t'$.

b) For the observer in S', the light does not appear to turn on simultaneously along the tube. Show that for the observer in S', the lighting up of the tube propagates with an apparent velocity $u = c^2/v$.

Question 3

Consider a light source (system S') and a light receiver (system S) approaching one another with a relative speed v. The length of the wave train in S is $L = c\Delta t - v\Delta t$, and it contains n wavelengths.

a) Derive the associated wavelength λ and frequency f.

b) Consider the situation in terms of the proper time of the moving source (S') and the corresponding frequency f_0. Derive the relation

$$f = \frac{\sqrt{1+\beta}}{\sqrt{1-\beta}}\, f_0.$$

This is the relativistic Doppler effect.

c) Assume now that the source and the receiver are moving away from each other with a relative speed v. What is the relation between the frequencies f and f_0 now?