TFY4345 Classical Mechanics. Department of Physics, NTNU.

ASSIGNMENT 8

Question 1



Figure 1: Pendulum with massive free support.

A massless rod with length ℓ has masses m_1 and m_2 at its two ends. The support (i.e., the mass m_1) can slide without friction along a horizontal constraint in the x direction. The system oscillates in a plane. Choose zero potential for m_1 and assume that the mass center does not move in the x direction.

- Find the Lagrangian of the system.
- Find an expression for the total energy E. Next, find an (integral) expression for the time $t = t(\theta)$. (You are not asked to solve this integral.)

Question 2

A rigid body rotates with angular velocity ω . In the lectures, we found the components of ω along the body axes x', y' and z' in terms of the Euler angles ϕ , θ and ψ . (Here, primed symbols for body axes.) Show that the components of ω along the axes fixed in space are

$$\begin{aligned}
\omega_x &= \dot{\theta} \cos \phi + \dot{\psi} \sin \theta \sin \phi \\
\omega_y &= \dot{\theta} \sin \phi - \dot{\psi} \sin \theta \cos \phi \\
\omega_z &= \dot{\psi} \cos \theta + \dot{\phi}
\end{aligned}$$

Question 3

You are sitting on a carousel in a seat located 5 m from the rotation axis. In your lap you hold a bag with mass 8 kg. Initially, the carousel is at rest, but then increases its angular velocity with a constant angular acceleration 0.2 rad/s² (counterclockwise). With what (horizontal) force \mathbf{F} do you hold your bag ten seconds after the carousel started rotating? Find both the absolute value and the direction of this force. (Set $g = 10 \text{ m/s}^2$.)



Figure 2: Carousel.