TFY4345 Classical Mechanics. Department of Physics, NTNU.

ASSIGNMENT 9

Question 1

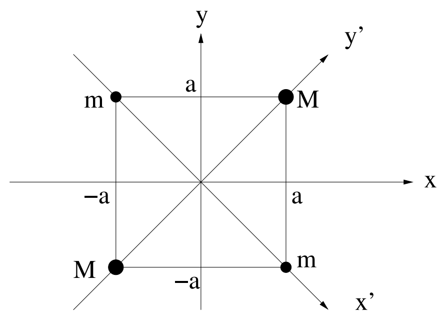

Figure 1: Four point masses in a plane.

For the four point masses in the figure above (all located in the xy plane), find the inertia tensor I_{jk}
a) in the coordinate system xyz
b) in the coordinate system $x'y'z'$

Question 2

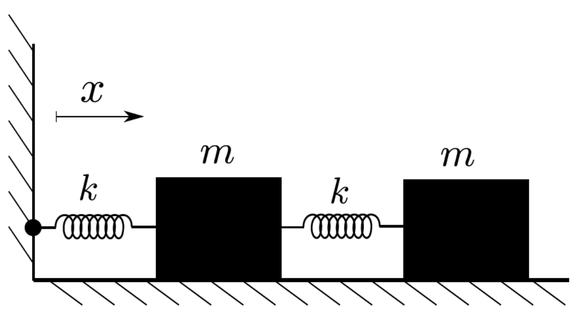

Figure 2: Two coupled oscillating masses.

Find the two normal modes (frequencies and relative amplitudes) for the horizontally oscillating masses in the figure above. (Neglect friction.) Let η_1 and η_2 denote the displacement from equilibrium for the left and right mass, respectively.

Question 3

In the lectures, we found the eigenfrequencies ω_α ($\alpha = 1, 2, 3$) for small oscillations along the molecular axis in a symmetric 3-atomic molecule (masses m, M and m and spring constants k between nearest neighbours). Derive the corresponding amplitude vectors A_α, where the components $A_{j\alpha}$ ($j = 1, 2, 3$) represent the oscillation amplitude (including sign) for atom j in normal mode α.

Question 4

Two inertial systems S and S' have a common origin at time $t = t' = 0$. The system S' moves with velocity $\boldsymbol{v} = v\hat{z}$ relative to S.
a) Use the Lorentz transformation (LT) to show that the equation for the light front for a light wave starting in $r = r' = 0$ at $t = t' = 0$ is described by the same equation in S and S'.
b) If S' has velocity $\boldsymbol{v} = c\boldsymbol{\beta}$ relative to S, show that the LT can be written as

$$\begin{aligned} \boldsymbol{r}' &= \boldsymbol{r} + (\gamma - 1)\frac{(\boldsymbol{\beta}\cdot\boldsymbol{r})\boldsymbol{\beta}}{\beta^2} - \gamma ct\boldsymbol{\beta} \\ t' &= \gamma t - \frac{\gamma}{c}\boldsymbol{\beta}\cdot\boldsymbol{r} \end{aligned}$$