## 1.1 Units.

a.) The four fundamental constants  $\hbar$  (Planck's constant), c (velocity of light),  $G_N$  (gravitational constant) and  $k_B$  (Boltzmann constant) can be combined to obtain the dimension of a length, time, mass, energy and temperature. Find the four relations and calculate the numerical values of two of them.

b.) Find the connection between a cross section  $\sigma$  expressed in units of cm², mbarn, and  ${\rm GeV^{-2}}.$ 

a.) A formal way to derive e.g. the Planck time  $t_{\rm Pl}$  is to solve

$$[c]^{\alpha}[\hbar]^{\beta}[G]^{\gamma}[k_{B}]^{\delta} = [cm/s]^{\alpha}[g\,cm^{2}/s]^{\beta}[cm^{3}/(g\,s^{2})]^{\gamma}[g\,cm^{2}/K]^{\delta} = s.$$

Simpler: i) note that  $k_B$  contains as only one the temperature T and enters therefore only  $T_{\text{Pl}}$  ii) we need a combination of  $\hbar G$  to cancel the gram.

iii) multiply with  $1/c^5$  to eliminate the centimeter,

$$[\hbar G/c^5] = \mathrm{s}^2$$

and thus

$$t_{\rm Pl} = \sqrt{\frac{\hbar G}{c^5}} \approx 5.4 \times 10^{-44} \mathrm{s} \,.$$

Then  $l_{\rm Pl} = ct_{\rm Pl} = 1.6 \times 10^{-33} \,\mathrm{cm}$ , etc.

The Planck units combine special relativity (they contain c), quantum effects (they contain  $\hbar$ ) and gravity (they contain G). This indicates that at energies  $E \gg M_{\rm Pl}c^2$ , lengths  $l \ll l_{\rm Pl}, \ldots$ , a quantum theory of gravity is required.