1.10 Transformation between inertial frames.
Consider two inertial frames K and K' with parallel axes at $t = t' = 0$ that are moving with the relative velocity v in the x direction.
a.) Show that the linear transformation between the coordinates in K and K' is given by

$$\begin{pmatrix} t' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} At + Bx \\ Dt + Ex \\ y \\ z \end{pmatrix} = \begin{pmatrix} At + Bx \\ A(x - vt) \\ y \\ z \end{pmatrix} \tag{14}$$

b.) Show that requiring the invariance of

$$\Delta s^2 \equiv c^2 t^2 - x^2 - y^2 - z^2 = c^2 t'^2 - x'^2 - y'^2 - z'^2 \tag{15}$$

leads to Lorentz transformations.
c.) What is the condition leading to Galilean transformations?

a.) The origin $x' = 0$ of K' correspond in K to $x = vt$. Then

$$0 = Dt + Evt \quad \Rightarrow \quad D = -Ev$$

The origin $x = 0$ of K correspond in K' to $x' = -vt'$. Then

$$t' = At \tag{16}$$

$$-vt' = Dt \quad \Rightarrow \quad t' = -\frac{D}{v} t = At \tag{17}$$

and thus $A = -D/v$ and hence $A = E$.

b.) Square the terms on the RHS of

$$t^2 - x^2 = (At - Bx)^2 - A^2(x - vt)^2 ,$$

order them as $t^2(\cdots) - x^2(\cdots) + 2tx(\cdots)$ and compare coefficients to the LHS. This gives $A = \gamma$ and $B = -\beta\gamma$.

c.) Absolute time $t = t'$ requires $A = 1$ and $B = 0$