1.11 Relativity of simultaneity.
Draw a space-time diagram (in $d = 2$) for two inertial frames connected by a boost with velocity β: What are the angles between the axes t and t', x and x'? Draw lines of constant t and t' and convince yourself that the time order of two space-like events is not invariant.

We parametrize a boost along the x direction by

$$\tilde{t} = t\cosh\eta + x\sinh\eta\,, \tag{18}$$

$$\tilde{x} = t\sinh\eta + x\cosh\eta\,, \tag{19}$$

with $\tilde{y} = y$ and $\tilde{z} = z$. Direct calculation using $\cosh^2(x) - \sinh^2(x) = 1$ shows that Δs^2 is invariant as desired.
Consider now in the system $\tilde{K}$ the origin of the system K. Then $x = 0$ and

$$\tilde{x} = t\sinh\eta \quad \text{and} \quad \tilde{t} = t\cosh\eta\,. \tag{20}$$

Dividing the two equations gives $\tilde{x}/\tilde{t} = \tanh\eta$. Since $\beta = \tilde{x}/\tilde{t}$ is the relative velocity of the two systems, we have identified the physical meaning of the imaginary "rotation angle η" as the rapidity

$$\eta = \operatorname{arctanh}(\beta)\,. \tag{21}$$

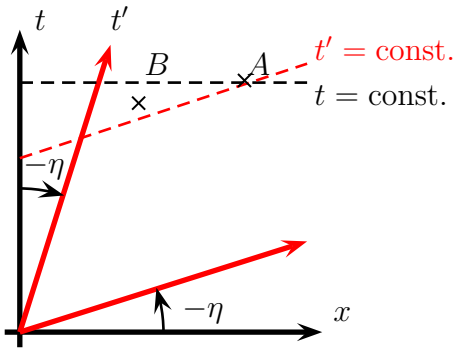
We obtain the lines of constant $\tilde{t}_0$ (i.e. the $\tilde{x}$ axis and its parallels) in the $x - t$ plane by solving (18) for t,

$$t = -\beta x + \tilde{t}_0 \tag{22}$$

Now we recall that a straight-line with $y = mx$ has the angle $\tan\alpha = m$ to the x axis. Thus the rotation angle in our case is given by η: Since $\beta \in [-1:1]$, $\tilde{x}$ axis has the angle between 0 and 45° with the x axis.
In the same way, the $\tilde{t}$ axis follows from (19) as

$$t = -\beta^{-1}x + \tilde{x}_0\,. \tag{23}$$

Now $1/\beta$ is in the range $[-\infty:1]$ and $[1:\infty]$; the $\tilde{t}$ axis has thus the angle between 0 and 45° to the t axis.



In the frame (t', x') moving with $\beta = \tanh(\eta)$ relative to the frame (t, x), the axes are rotated by η. Space-time events like B that are inbetween $t = const$ and $t' = const$ have a different time order relative to A: $t'_B > t'_A$ and $t_A > t_B$. Since $|\eta| < 45°$, time-like events are time-ordered.
Note that the t', x' axes are orthogonal to each other (in space-time), although they are not plotted in the Euclidean geometry of $\mathbb{R}^2$.