10.8 Gauge boson propagator in axial gauge.
The axial gauge condition is $n^\mu A^a_\mu = 0$ where n is a fixed vector. a.) Show that the Fadeev-Popov term is independent of A^a_μ and, thus, can be absorbed in the normalisation of the path integeral. b.) Derive the gauge boson propagator using

$$\mathscr{L}_{\rm gf} = \frac{1}{2\xi}(n^\mu A_\mu)^2$$

as the corresponding gauge-fixing term.

missing: c.) Derive the gauge boson propagator in Coulomb gauge and split the result into a transverse, longitudinal and Coulomb part.

0.) It may be useful to recall the case of the Maxwell equations. Choosing in $\partial_\mu F^{\mu\nu} = \partial_\mu(\partial^\mu A^\nu - \partial^\nu A^\mu) = \Box A^\nu - \partial_\mu \partial^\nu A^\mu = j^\nu$ a covariant gauge as e.g. the Lorenz gauge $\partial_\mu A^\mu = 0$ reduces the d.o.f. by one to three. We can still add to the potential A^μ any $\partial^\mu \chi$ satisfying $\Box\chi = 0$, setting e.g. $A^0 = 0$, giving $\boldsymbol{\nabla} \cdot \boldsymbol{A} = 0$. Now we have only 2 d.o.f., but explicit Lorentz invariance is lost. This choice is with $n^\mu = (1,0,0,0)$ a special case of the axial gauges.

a.) We have to evaluate

$$\det\left(\frac{\delta g^a}{\delta \vartheta^b}\right)\prod_{x,a}\delta(g^a)$$

for the gauge condition $g^a = n^\mu A^a_\mu = 0$. An infinitesimal gauge transformation leads to

$$\delta g^a = \delta(n^\mu A^a_\mu) = -n^\mu D^{ac}_\mu \vartheta^c = n^\mu(\delta^{ac}\partial_\mu - g f^{abc} A^b_\mu)\vartheta^c = n^\mu \partial_\mu \vartheta^a\,,$$

where we used $n^\mu A^a_\mu = 0$ in the last step. Thus the Fadeev-Popov Lagrangian is independent of the gauge fields,

$$\mathscr{L}_{\rm FP} = -\bar{c}\frac{\partial G}{\partial \vartheta} c = -\bar{c} n^\mu \partial_\mu c\,,$$

and changes only the normalisation of the generating functional.

b.) We have to consider only the quadratic terms, thus the non-abelian terms play role, and we suppress the group index a.

$$\mathscr{L}_{\rm eff} = \mathscr{L}_{\rm cl} + \mathscr{L}_{\rm gf} = \frac{1}{2}A_\mu(\eta^{\mu\nu}\Box - \partial^\mu\partial^\nu)A_\nu + \frac{1}{2\xi}A_\mu n^\mu n^\nu A_\nu = \frac{1}{2}A_\mu P^{\mu\nu} A_\nu\,.$$

The propagator $D^{\mu\nu}$ has to satisfy

$$P_{\mu\nu} D^{\nu\lambda}(x-y) = \delta^\lambda_\mu \delta(x-y)\,.$$

FT gives $P_{\mu\nu}(k) D^{\nu\lambda}(k) = \delta^\lambda_\mu$ with

$$P^{\mu\nu}(k) = -\eta^{\mu\nu}k^2 + k^\mu k^\nu + \frac{1}{\xi}n^\mu n^\nu\,.$$

Plugging the tensor decomposition

$$D^{\mu\nu}(k) = A\eta^{\mu\nu} + B(n^\mu k^\nu + n^\nu k^\mu) + Ck^\mu k^\nu + Dn^\mu n^\nu$$

into the ansatz and comparing the coefficients leads to

$$A = -\frac{1}{k^2}, \qquad B = -\frac{1}{nk}\frac{1}{k^2}, \qquad C = -\frac{n^2 - \xi k^2}{(nk)^2 k^2}, \quad \text{and} \quad D = 0\,.$$

Thus the gauge boson propagator in axial gauge is given by

$$D^{\mu\nu}(k) = \frac{1}{k^2}\left[-\eta^{\mu\nu} + \frac{1}{nk}(n^\mu k^\nu + n^\nu k^\mu) - \frac{n^2 - \xi k^2}{(nk)^2 k^2} k^\mu k^\nu\right]\,.$$