

11.2 DR and short distance physics.

Our usual regularisation methods are based on modifying the short-distance (=UV) behavior. Discuss what happens in DR for the typical integral

$$I(\omega) = \int \frac{d^{2\omega}k}{(2\pi)^{2\omega}} \frac{1}{[k^2 + M^2 + i\varepsilon]^\alpha} \quad (209)$$

where $2\omega = d = 4 - 2\varepsilon$. The parameter M contains the dependence on masses m_i and external momenta p . Perform the integral over the -2ε extra dimensions and rewrite $I(\omega)$ as

$$I(\omega) = c(\varepsilon) \int \frac{d^4k}{(2\pi)^4} \frac{1}{[k^2 + M^2 + i\varepsilon]^\alpha} R(\varepsilon). \quad (210)$$

What is the behavior of the regulator $R(\varepsilon)$?

We split the integral into the usual 4-dimensional part and a piece containing the additional -2ε dimensions,

$$I(\omega) = \int \frac{d^{-2\varepsilon}k}{(2\pi)^{-2\varepsilon}} \frac{d^4k}{(2\pi)^4} \frac{1}{[k^2 + k_\varepsilon^2 + M^2 + i\varepsilon]^\alpha}. \quad (211)$$

Adding the mass scale $\mu^{2\varepsilon}$ and performing the integral over -2ε extra dimensions, we obtain

$$I(\omega) = \frac{\Gamma(\alpha + \varepsilon)}{\Gamma(\alpha)} \int \frac{d^4k}{(2\pi)^4} \frac{1}{[k^2 + M^2 + i\varepsilon]^\alpha} \left(\frac{k^2 + M^2}{4\pi\mu^2} \right)^{-\varepsilon}. \quad (212)$$

The prefactor goes to one for $\varepsilon \rightarrow 0$. The final factor

$$R(\varepsilon) \equiv \rho^{-\varepsilon} \equiv \left(\frac{k^2 + M^2}{4\pi\mu^2} \right)^{-\varepsilon} \quad (213)$$

plays the role of a regulator, with

$$\rho^{-\varepsilon} = e^{-\varepsilon \ln \rho} \rightarrow 1 \quad (214)$$

for $|\ln \rho| \ll 1/\varepsilon$. Thus physics is unchanged, if the loop momentum k and the external momenta and masses are of order μ . If however k or M are much larger than μ such that $\ln \rho \gg 1/\varepsilon$, then the factor $\rho^{-\varepsilon}$ will improve the convergence of the integral—thus short distance physics is modified as expected.

Note that i) physics is also changed in the opposite limit, i.e. when k and M are much smaller than μ . If particles are massive, then M^2 is bounded from below and no problem arises. Second, going to $2\omega = d = 4 + 2\varepsilon$, i.e. changing the sign of ε , we can use DR to regularize IR divergencies.