**11.3 Dimensional regularisation and $\gamma^5$.**
Show that the properties $\mathrm{tr}(\gamma^\mu\gamma^\nu) = d\eta^{\mu\nu}$, $\mathrm{tr}(1) = 4$ and $\{\gamma^\mu,\gamma^5\} = 0$ lead to an inconsistency in $d \neq 4$ dimensions. (Hint: Consider first $d\,\mathrm{tr}[\gamma^5] = -d\,\mathrm{tr}[\gamma^5]$, then $d\,\mathrm{tr}[\gamma^5\gamma_\alpha\gamma_\beta] = (4-d)\mathrm{tr}[\gamma^5\gamma_\alpha\gamma_\beta]$ and finally $(4-d)\mathrm{tr}\left[\gamma^5\gamma_\alpha\gamma_\beta\gamma_\rho\gamma_\sigma\right] = 0$.]

We show that $\{\gamma^\mu,\gamma^\nu\} = 2\eta^{\mu\nu}\mathbf{1}_d$ with $\mathrm{tr}(\mathbf{1}_d) = f(d)$ and $f(4) = 4$ is incompatibel with $\{\gamma^\mu,\gamma^5\} = 0$.
Setting $f(d) = 4$, it is

$$\gamma_\mu\gamma^\mu = \frac{1}{2}\{\gamma_\mu,\gamma^\mu\} = d\,.$$

Next we consider

$$d\,\mathrm{tr}[\gamma^5] = \mathrm{tr}[\gamma^5\gamma_\mu\gamma^\mu] = -\mathrm{tr}[\gamma^5\gamma_\mu\gamma^\mu] = -d\,\mathrm{tr}[\gamma^5]$$

and thus $\mathrm{tr}(\gamma^5) = 0$ (except possibly for $d = 0$). Analyticity implies then $\mathrm{tr}(\gamma^5) = 0$ for all $d$.

Then we consider

$$d\,\mathrm{tr}[\gamma^5\gamma_\mu\gamma_\nu] = \mathrm{tr}[\gamma^5\gamma_\mu\gamma_\nu\gamma_\lambda\gamma^\lambda] = -\mathrm{tr}[\gamma^5\gamma^\lambda\gamma_\mu\gamma_\nu\gamma_\lambda]$$

Now we use twice $\gamma^\lambda\gamma_\mu = 2\delta^\lambda_\mu - \gamma_\mu\gamma^\lambda$ to find

$$d\,\mathrm{tr}[\gamma^5\gamma_\mu\gamma_\nu = -2\delta^\lambda_\mu\mathrm{tr}[\gamma^5\gamma_\nu\gamma_\lambda] + 2\delta^\lambda_\nu\mathrm{tr}[\gamma^5\gamma_\mu\gamma_\lambda] - d\mathrm{tr}[\gamma^5\gamma_\mu\gamma_\nu] \tag{215}$$

$$= 4\mathrm{tr}[\gamma^5\gamma_\mu\gamma_\nu] - d\mathrm{tr}[\gamma^5\gamma_\mu\gamma_\nu] \tag{216}$$

and thus $(2-d)\mathrm{tr}(\gamma^5\gamma_\mu\gamma_\nu) = 0$ or $\mathrm{tr}(\gamma^5\gamma_\mu\gamma_\nu) = 0$ (except possibly for $d = 2$). Analyticity implies again $\mathrm{tr}(\gamma^5\gamma_\mu\gamma_\nu) = 0$ for all $d$.

Procedding in the same way for 4 gammas results in

$$(4-d)\mathrm{tr}(\gamma^5\gamma_\mu\gamma_\nu\gamma_\lambda\gamma_\kappa) = 0$$

If we require analyticity (needed to do DR), the trace is zero – in contradiction to our $d = 4$ result.