12.10 Asymptotic freedom and scalar theories.

In QCD, the dominance of the bosonic loop contributions to the beta function leads to asymptotic freedom. Which difference is responsible that the scalar $\lambda \phi^4$ theory is in contrast not asymptotically free?

In a gauge theory, the Ward identities imply that

$$g(\mu) = \frac{Z_2 Z_3^{1/2}}{Z_1} g_0 = Z_3^{1/2} g_0.$$

In contrast, in the ϕ^4 theory (at one loop level)

$$\lambda(\mu) = \frac{Z_{\phi}^2}{Z_{\lambda}} \lambda_0 = Z_{\lambda}^{-1/2} \lambda_0 \,.$$

Thus in one case the renormalisation constant is in the nominator, in the other in the denominator. When we expand for small ε , we get an additional minus sign in case of the ϕ^4 theory.