15.4 Pressure integral.

a.) Comparing the 1. law of thermodynamics, $\mathrm{d}U = T\mathrm{d}S - P\mathrm{d}V$, with the total differential

$$\mathrm{d}U = (\partial U/\partial S)_V \mathrm{d}S + (\partial U/\partial V)_S \mathrm{d}V$$

gives $P = -(\partial U/\partial V)_S$.

b.) Since

$$U = V \int \frac{\mathrm{d}^3 p}{(2\pi)^3}\, E f(p)$$

and

$$S \propto \ln(V f(p))$$

differentiating U keeping S constant means

$$P = -V \int \frac{\mathrm{d}^3 p}{(2\pi)^3}\, (\partial E/\partial V) f(p)\,.$$

We write $\partial E/\partial V = (\partial E/\partial p)(\partial p/\partial L)(\partial L/\partial V)$. To evaluate this we note that $\partial E/\partial p = p/E$, that from $V = L^3$ it follows $\partial L/\partial V = 1/(3L^2)$ and that finally the quantization conditions of free particles, $p_k = 2\pi k/L$ implies $\partial p/\partial L = -p/L$. Combined this gives $\partial E/\partial V = -p^2/(3EV)$.