## Selected Solutions

## 16.6 Derrick's theorem.

The energy functional is

$$E[\phi] = T[\phi] + V[\phi] = \int d^{n}x \, \frac{1}{2} (\partial_{i}\phi)^{2} + \int d^{n}x \, V(\phi) \,.$$
(271)

Under a rescaling  $x \to \tilde{x} = x/\lambda$ , the kinetic energy changes with

$$\frac{\partial}{\partial x}\phi(\tilde{x}) = \frac{\partial}{\partial \tilde{x}}\frac{\partial \tilde{x}}{\partial x}\phi(\tilde{x}) = \frac{1}{\lambda}\frac{\partial\phi(\tilde{x})}{\partial \tilde{x}}$$
(272)

as

$$T_{\lambda}[\phi] = \frac{1}{2} \int \mathrm{d}^{n} x \, \left(\frac{\partial}{\partial x_{i}} \, \phi(\tilde{x})\right)^{2} = \frac{\lambda^{n-2}}{2} \int \mathrm{d}^{n} \tilde{x} \, \left(\frac{\partial}{\partial \tilde{x}_{i}} \, \phi(\tilde{x})\right)^{2} = \lambda^{n-2} T[\phi]. \tag{273}$$

while the potential energy scales as

$$V_{\lambda}[\phi] = \int \mathrm{d}^{n} x \, V(\phi(x/\lambda)) = \lambda^{n} \int \mathrm{d}^{n} \tilde{x} \, V(\phi(\tilde{x})) = \lambda^{n} V[\phi] \,. \tag{274}$$

Thus

$$E(\lambda) = \lambda^{n-2}T[\phi] + \lambda^n V[\phi]$$
(275)

If the initial solution for  $\lambda = 1$  is stable, then we should have E'(1) = 0 and E''(1) < 0. But

$$E'(\lambda) = (n-2)\lambda^{n-3}T + n\lambda^{n-1}V$$
(276)

$$E'(1) = (n-2)T + nV.$$
(277)

Since both T and V are positive definite, stable solutions are only possible for n = 1.