19.3 Cosmological constant Λ ♣.

a.) In Minkowski space, the stress tensor of an ideal fluid is $T_{\mu\nu} = \mathrm{diag}\{\rho, P, P, P\}$ in an inertial system where the fluid is at rest. Comparing this to $\kappa T_{\mu\nu} = \Lambda \eta_{\mu\nu}$, it follows $\rho_\Lambda \equiv \Lambda/\kappa = -P_\Lambda$ or $w = -1$.

b.) In thermodynamics, the pressure P is defined by $P = -(\partial U/\partial V)_S$. With $U_\Lambda = \rho_\Lambda V$ and $\rho_\Lambda = \text{const.}$ it follows $P_\Lambda = -\rho_\Lambda$.

c.) Λ has the dimension length^{-2}. For an estimate, we can ask that the curvature radius $\Lambda^{-1/2}$ is larger than 3000 Mpc. This corresponds to an energy density

$$|\rho_\Lambda| \lesssim \frac{\Lambda}{8\pi G} = \frac{\Lambda c^4}{8\pi G} \sim 6 \times 10^{-9} \frac{\text{erg}}{\text{cm}^3} \sim 4 \times 10^{-6} \frac{\text{GeV}}{\text{cm}^3}$$

where we inserted in the second step a factor c^4 to obtain the correct units. This corresponds aproximately to the density of a flat universe with the observed expansion velocity. (This density is the so-called critical density which we will define in chapter 20).