2.3 Classical limit.
Sketch (without detailed calculation) why in the path integral the allowed paths dominate in the classical limit.

Sketch: If the phase oscillates fast, contributions of nearby paths will interfere destructively. Only exception are classically allowed paths: Since then $S[q]$ is stationary, they will add up and dominate the path integral.

More qualitatively: Consider two close paths $q(t)$ and $\tilde{q}(t) = q(t) + \eta(t)$. Then

$$S[\tilde{q}] = S[q + \eta] = S[q] + \int \mathrm{d}t\, \eta(t) \frac{\delta S[q]}{\delta q(t)} + \mathcal{O}(\eta^2)\,.$$

Adding their contribution to the amplitude gives

$$A \simeq \mathrm{e}^{\mathrm{i}S[q]/\hbar} \left(1 + \exp \frac{\mathrm{i}}{\hbar} \int \mathrm{d}t\, \eta(t) \frac{\delta S[q]}{\delta q(t)} \right)\,.$$

The difference $\Delta S/\hbar$ determines, if the two contributions add up or cancel: In the classical limit, $\Delta S \gg \hbar$, and in general destructive interference happens even for close-by paths. However, if $q(t)$ is classically allowed, then

$$S[\tilde{q}] = S[q] + \mathcal{O}(\eta^2)$$

and nearby paths can sum up. In the quantum limit, $\Delta S \ll \hbar$, and there's always a complicated interference of paths.