20.10 Temperature-time relation.

We use the Friedmann equation for a radiation-dominated universe, $H^2 = 8\pi/3G\rho$, together $\rho = g\pi^2 T^4/30$,

$$H^2 = g\frac{8\pi}{90}GT^4 = \left(\frac{1}{2t}\right)^2$$

Solving for t

$$t = \sqrt{\frac{90}{32\pi^3 gG}}\frac{1}{T^2}$$

and inserting constants

$$\frac{t}{\mathrm{s}} = \sqrt{\frac{45(1.2\times10^{19}\mathrm{GeV})^2}{16g\pi^3\mathrm{s}^2\mathrm{MeV}^4}}\left(\frac{\mathrm{MeV}}{T}\right)^2$$

and using that $s^{-1} = 6.6\times10^{-25}$ GeV, it follows

$$\frac{t}{\mathrm{s}} = \sqrt{\frac{45g(6.6\times10^{-25}\mathrm{GeV})^2(2.4\times10^{18}\mathrm{GeV})^2}{16\pi^3 10^{-12}\mathrm{GeV}^4}}\left(\frac{\mathrm{MeV}}{T}\right)^2 \approx \frac{2.4}{\sqrt{g}}\left(\frac{\mathrm{MeV}}{T}\right)^2.$$

Thus at $t = 1\,\mathrm{s}$, the temperature is close to the decoupling of e^-e^+. Below it is $g_* = 29/4$, above $g_* = 43/4$. Instering $g_* = 43/4$, it follows $T = 1.2\,\mathrm{MeV}$.