## 20.14 Steady-state theory

a.) The Hubble flow induces the flux vn through the surface  $4\pi r^2$  of a sphere with radius r,  $\dot{N}_- = 4\pi r^2 vn$ . Inside the sphere,  $\dot{N}_+ = 4\pi/3r^3\dot{n}$  particles are generated. The two effects should balance each other,  $N_+ = N_-$ , thus  $\dot{n} = 3vn/r = 3Hn \approx 3 \times 0.7/9.8 \,\text{Gyr} \times 8/\text{m}^3 \approx 2$  H-atoms per Gyr and  $m^3$ .

b.) From Hubble's law, v = Hd or  $\dot{R} = HR$ , and H = const., it follows  $R(t) = R_0 \exp(Ht)$ . The exponential function is self-similar, and as expected an observer is not able to ascribe to t an absolute meaning.

c.) To keep number of galaxies constant, newly created matter has to form galaxies. At a given epoch,  $t_0$ , the mean number of galaxies in the volume V with ages in the range t to t + dt is the mean number that were created in the same comoving volume the time t earlier,

$$dn_g = dt \, 3Hn_g \, V[R(t_0 - t)/R(t)]^3$$

Thus the distribution of galaxy ages follows as

$$\frac{dn_g}{dt} = 3Hn_g \exp(-3H_0 t), \qquad n_g = n_0 \exp(-3H_0 t)$$

and the mean age of a galaxy is  $\langle t \rangle = 1/(3H_0) \sim 3$  Gyr.