

26.1 Smooth cutoff.

Recalculate ρ and P using a smooth exponential cut-off instead of a sharp cut-off (for simplicity, you may consider only the case $m = 0$).

Adding as exponential cut-off $\exp(-ak)$ with $a = 1/\Lambda$ gives

$$\langle \rho \rangle = \frac{1}{4\pi^2} \int_0^\infty dk k^3 \exp(-ak) = \frac{1}{4\pi^2} \frac{\Gamma(4)}{a^4} = \frac{6\Lambda^4}{4\pi^2}, \quad (339)$$

i.e. again a quartic divergence. Calculating also $\langle \rho \rangle$ confirms $w = 1/3$.