4.1 $Z[J]$ at order λ.

Calculating $\delta F[J]/\delta J(x)$ for $F[J] = f\left(W_0[J]\right)$

$$\frac{\delta F[J]}{\delta J(x)} = \lim_{\epsilon\to 0}\frac{1}{\epsilon} f(W_0[\phi(x)+\epsilon\delta(x-y)]) - f(W_0[\phi(x)]) \tag{78}$$

$$= \lim_{\epsilon\to 0}\frac{1}{\epsilon} f(W_0[\phi(x)] + \epsilon\frac{\delta W_0}{\delta\phi}) - f(W_0[\phi(x)]) \tag{79}$$

$$= \lim_{\epsilon\to 0}\frac{1}{\epsilon} f(W_0[\phi(x)]) + g'\epsilon\frac{\delta W_0}{\delta\phi} - f(W_0[\phi(x)]) \tag{80}$$

$$= f'(W_0[J])\frac{\delta W_0}{\delta J} \tag{81}$$

Calculating the first derivative

$$\frac{\delta}{\mathrm{i}\delta J(x)}\exp\left(\mathrm{i}W_0[J]\right) = \frac{\delta W_0[J]}{\delta J(x)}\exp\left(\mathrm{i}W_0[J]\right) \tag{82}$$

Calculating the second derivative (using the functional derivative product rule)

$$\left(\frac{\delta}{\mathrm{i}\delta J(x)}\right)^2\exp\left(\mathrm{i}W_0[J]\right) = \left(\left(\frac{\delta W_0[J]}{\delta J(x)}\right)^2 + \frac{1}{i}\frac{\delta^2 W_0[J]}{\delta J(x)^2}\right)\exp\left(\mathrm{i}W_0[J]\right) \tag{83}$$

Calculating the third derivative

$$\left(\frac{\delta}{\mathrm{i}\delta J(x)}\right)^3\exp\left(\mathrm{i}W_0[J]\right) = \left(\left(\frac{\delta W_0[J]}{\delta J(x)}\right)^3 + \frac{3}{i}\frac{\delta^2 W_0[J]}{\delta J(x)^2}\frac{\delta W_0[J]}{\delta J(x)} + \frac{1}{i^2}\frac{\delta^3 W_0[J]}{\delta J(x)^3}\right)\exp\left(\mathrm{i}W_0[J]\right) \tag{84}$$

Calculating the fourth derivative

$$\left(\frac{\delta}{\mathrm{i}\delta J(x)}\right)^4\exp\left(\mathrm{i}W_0[J]\right) = \left(\left(\frac{\delta W_0[J]}{\delta J(x)}\right)^4 + \frac{6}{i}\frac{\delta^2 W_0[J]}{\delta J(x)^2}\left(\frac{\delta W_0[J]}{\delta J(x)}\right)^2 + \frac{3}{i^2}\left(\frac{\delta^2 W_0[J]}{\delta J(x)^2}\right)^2 + \right.$$
$$\left. + \frac{4}{i^2}\frac{\delta W_0[J]}{\delta J(x)}\frac{\delta^3 W_0[J]}{\delta J(x)^3} + \frac{1}{i^3}\frac{\delta^4 W_0[J]}{\delta J(x)^4}\right)\exp\left(\mathrm{i}W_0[J]\right)$$
$$= \left(\left(\frac{\delta W_0[J]}{\delta J(x)}\right)^4 + \frac{6}{i}\frac{\delta^2 W_0[J]}{\delta J(x)^2}\left(\frac{\delta W_0[J]}{\delta J(x)}\right)^2 + \frac{3}{i^2}\left(\frac{\delta^2 W_0[J]}{\delta J(x)^2}\right)^2\right)\exp\left(\mathrm{i}W_0[J]\right)$$

Substituting the functional derivatives

$$\left(\frac{\delta}{\mathrm{i}\delta J(x)}\right)^4\exp\left(\mathrm{i}W_0[J]\right) = \left[\left(\int d^4y\Delta_F(y-x)J(y)\right)^4 + 6i\Delta_F(0)\left(\int d^4y\Delta_F(y-x)J(y)\right)^2\right.$$
$$\left. + 3\left(i\Delta_F(0)\right)^2\right]\exp\left(\mathrm{i}W_0[J]\right)$$