4.9 Feynman integrals.
Derive the relation (4.98).

As example, we want to check the relation $I_\mu(\omega,\alpha) = -p_\mu I(\omega,\alpha)$. We start differentiating in

$$I(\omega,\alpha) = \int \frac{\mathrm{d}^{2\omega}k}{(2\pi)^{2\omega}} \frac{1}{[k^2+2pk-m^2+\mathrm{i}\varepsilon]^\alpha} = \mathrm{i}\,\frac{(-1)^\alpha}{(4\pi)^\omega}\,\frac{\Gamma(\alpha-\omega)}{\Gamma(\alpha)}\,[m^2+p^2-\mathrm{i}\varepsilon]^{\omega-\alpha}\,. \tag{106}$$

on the LHS

$$\frac{\partial}{\partial p^\mu}\frac{1}{[k^2+2pk-m^2+\mathrm{i}\varepsilon]^\alpha} = \frac{-2\alpha k^\mu}{[k^2+2pk-m^2+\mathrm{i}\varepsilon]^{\alpha+1}} \tag{107}$$

and on the RHS,

$$\frac{\partial}{\partial p^\mu}[m^2+p^2-\mathrm{i}\varepsilon]^{\omega-\alpha} = 2p^\mu\,(\omega-\alpha)[m^2+p^2-\mathrm{i}\varepsilon]^{\omega-\alpha+1}\,. \tag{108}$$

Comparing both sides, we find

$$I_\mu(\omega,\alpha+1) = \int \frac{\mathrm{d}^{2\omega}k}{(2\pi)^{2\omega}} \frac{k^\mu}{[k^2+2pk-m^2+\mathrm{i}\varepsilon]^{\alpha+1}} \tag{109}$$

$$= p^\mu\,\frac{\alpha-\omega}{\alpha}\,\mathrm{i}\,\frac{(-1)^\alpha}{(4\pi)^\omega}\,\frac{\Gamma(\alpha-\omega)}{\Gamma(\alpha)}\,[m^2+p^2-\mathrm{i}\varepsilon]^{\omega-\alpha+1} \tag{110}$$

Next we set $\tilde\alpha = \alpha+1$,

$$I_\mu(\omega,\tilde\alpha) = -p^\mu\,\mathrm{i}\,\frac{(-1)^{\tilde\alpha}}{(4\pi)^\omega}\,\frac{\tilde\alpha-1-\omega}{\tilde\alpha-1}\,\frac{\Gamma(\tilde\alpha-1-\omega)}{\Gamma(\tilde\alpha-1)}\,[m^2+p^2-\mathrm{i}\varepsilon]^{\omega-\tilde\alpha} \tag{111}$$

Now we use $(z-1)\Gamma(z-1) = \Gamma(z)$, forget the tilde, and obtain $I_\mu(\omega,\alpha) = -p_\mu I(\omega,\alpha)$.