5.1 Scalar fields.

The products $\phi_i\phi_j$ and $\partial^\mu\phi_i\partial_\mu\phi_j$ are symmetric, and thus a possible antisymmetric part of A and B drops pout. Conservation of probability requires that the Hamiltonian density $\mathscr{H}$ is Hermitain. Thus A, B and C should be real. The energy should bounded from below, and thus A and B are (semi-) positiv definite matrices: Their eigenvalues are ≥ 0 and thus we can form $A^{1/2}$.
Then we redefine the fields as

$$\tilde{\phi}_i = (A^{-1/2}\phi)_i$$

which makes the kinetic term diagonal, while the mass term becomes

$$\tilde{B}_{ij} = (A^{-1/2}BA^{-1/2})_{ij}\,.$$

Next we diagonalise B by an orthogonal transformation, $\phi' = O\tilde{\phi}$, and $O^T\tilde{B}O = \mathrm{diag}(b_1, \ldots, b_n)$ with $b_i \equiv m_i^2 \geq 0$.
Note that it is sufficient to consider only linear transformations which maintain second-order equations of motion.