5.8 Local U(1) transformation.

Only problematic term is the kinetic energy $\mathscr{L} = -\phi^*\Box\phi$. For an infinitesimal transformation, $\phi \to \tilde{\phi} = (1 + \mathrm{i}\alpha)\phi$ we find

$$\Box\tilde{\phi} = \partial^\mu\partial_\mu\tilde{\phi} = \partial^\mu(\partial_\mu\phi + \mathrm{i}\alpha\partial_\mu\phi + \mathrm{i}\partial_\mu\alpha\phi) = \tag{141}$$

$$= \Box\phi + \mathrm{i}\Box\alpha\phi + 2\mathrm{i}\partial^\mu\alpha\partial_\mu\phi + \mathrm{i}\alpha\Box\phi\,. \tag{142}$$

Multiplying with $\tilde{\phi}^* = (1 - \mathrm{i}\alpha)\phi^*$, the $\alpha\phi^*\Box\phi$ terms cancel and we are left in linear order with

$$\tilde{\phi}\Box\tilde{\phi} = \phi^*\Box\phi + \mathrm{i}\Box\alpha\phi^*\phi + 2\mathrm{i}\partial^\mu\alpha\partial_\mu\phi + \mathcal{O}(\alpha^2)\,. \tag{143}$$

Thus the Lagrangian changes as

$$\delta\mathscr{L} = \mathrm{i}\,[2\partial^\mu\alpha\phi^*\partial_\mu\phi + \Box\alpha\phi^*\phi] \tag{144}$$

$$= \mathrm{i}\partial^\mu\alpha\,[\phi^*\partial_\mu\phi - \partial_\mu\phi^*\phi] = \partial^\mu\alpha j_\mu\,, \tag{145}$$

where we performed a partial integration and used the product rule.

If one prefers the form $\mathscr{L} = \partial_\mu\phi^*\partial^\mu\phi$ for the kinetic energy, then

$$\partial_\mu\tilde{\phi}^*\partial^\mu\tilde{\phi} = (\partial_\mu\phi^* - \mathrm{i}\alpha\partial_\mu\phi^* - \mathrm{i}\partial_\mu\alpha\phi^*)(\partial^\mu\phi + \mathrm{i}\alpha\partial^\mu\phi + \mathrm{i}\partial^\mu\alpha\phi) = \tag{146}$$

$$= \partial_\mu\phi^*\partial^\mu\phi + \mathrm{i}\partial_\mu\phi^*(\alpha\partial^\mu\phi + \partial^\mu\alpha\phi) - \mathrm{i}(\alpha\partial_\mu\phi^* + \partial_\mu\alpha\phi^*)\partial^\mu\phi = \tag{147}$$

$$= \partial_\mu\phi^*\partial^\mu\phi + \mathrm{i}\partial_\mu\phi^*\partial^\mu\alpha\phi - \mathrm{i}\partial_\mu\alpha\phi^*\partial^\mu\phi = \tag{148}$$

$$= \partial_\mu\phi^*\partial^\mu\phi - \partial^\mu\alpha j_\mu \tag{149}$$

leads to the same $\delta\mathscr{L}$.