7.4 Physical states of massless particles.

a.) The polarisation states of massless particles propagating with $\boldsymbol{k} \| \boldsymbol{e}_{d-1}$ are in $d = 5$

$$\varepsilon^{\mu} = \begin{pmatrix} 0 \\ \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ 0 \end{pmatrix} \quad \text{and} \quad \varepsilon^{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} & 0 \\ 0 & \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} & 0 \\ 0 & \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{163}$$

for spin $s = 1$ and $s = 2$, respectively. Thus the photon has 3 polarisation states. For the graviton, $\varepsilon^{\mu\nu} = \varepsilon^{\nu\mu}$ and $\varepsilon^{\nu}_{\nu} = 0$ implies that there are $1 + 2 + 3 - 1 = 5$ states.

b.) The photon has $d - 2$ polarisation states. For the graviton, we note first that a symmetric $m \times m$ matrix has $\sum_{n=1}^{m} n = m(m + 1)/2$ independent entries. Thus the graviton has with $m = d - 2$

$$\frac{(d-2)(d-1)}{2} - 1 = \frac{d(d-3)}{2}$$

states. In particular, we see that gravitons (and thus in th classical linit gravitational waves) exist only for $d \geq 4$, while for $d = 26$ the graviton has 299 polarisation states.