8.3 Derivation of the Dirac equation.

Expand $\exp\left[\frac{\boldsymbol{\sigma\eta}}{2}\right]$ and split the sum into even and odd terms, using $(\boldsymbol{\sigma\eta})^{2n} = 1$ and $(\boldsymbol{\sigma\eta})^{2n+1} = \boldsymbol{\sigma\eta}$.

Insert $\gamma = E/m$, multiply with $[(E+m)/(E+m)]^{1/2}$ and combine the two term.

Set $\phi_L(0) = \phi_R(0)$ and eliminate the zero momentum spinor,

$$\phi_R(p) = \frac{E + m + \boldsymbol{\sigma p}}{\sqrt{2m(E+m)}}\,\phi_R(0) = \frac{E + m + \boldsymbol{\sigma p}}{E + m - \boldsymbol{\sigma p}}\,\phi_L(p) = A\,\phi_L(p) \tag{180}$$

Multiply A with $(E + m - \boldsymbol{\sigma p})/(E + m - \boldsymbol{\sigma p})$ and evaluate the expression

$$A = \frac{(E+m)^2 + \boldsymbol{p}^2 + 2(E+m)\boldsymbol{\sigma p}}{(E+m)^2 - \boldsymbol{p}^2)} = \frac{2E^2 + 2Em + 2(E+m)\boldsymbol{\sigma p}}{2m^2 + 2Em} = \frac{E + \boldsymbol{\sigma p}}{m}\,. \tag{181}$$