9.4 Identities for gamma matrices.
Calculate using e.g. the tensor method

$$\mathrm{tr}[\not{a}\not{b}]$$

$$\mathrm{tr}[\not{a}\not{b}\not{c}\not{d}]$$

$$\gamma^\mu \not{a} \gamma_\mu$$

Using the tensor method, we have to express $\mathrm{tr}[\gamma^\mu \cdots \gamma^\nu \cdots (\gamma^5)]$ as the sum of possible combinations of allowed tensors ($\eta^{\mu\nu}$ and possibly $\varepsilon^{\mu\nu\lambda\sigma}$) taking into account the cyclic property of the trace.
Contracting $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$ with $\eta_{\mu\nu}$ gives

$$2\gamma^\mu\gamma_\mu = 2\eta^\mu_\mu = 8$$

or $\gamma^\mu\gamma_\mu = 4$. Thus

$$\gamma^\lambda\gamma_\mu\gamma_\lambda = \gamma^\lambda(2\eta_{\mu\lambda} - \gamma_\lambda\gamma_\mu) = 2\gamma_\mu - 4\gamma_\mu = -2\gamma_\mu$$

or $\gamma^\mu \not{a} \gamma_\mu = -2\not{a}$.

We determine next $\mathrm{tr}(\gamma^\mu\gamma^\nu)$ using that this expression has to be proportional to the metric tensor, $\mathrm{tr}(\gamma^\mu\gamma^\nu) = A\eta^{\mu\nu}$. Contraction gives

$$\mathrm{tr}(4 \cdot 1) = 4A$$

or $A = 4$ and $\mathrm{tr}[\not{a}\not{b}] = 4a \cdot b$.
Similarly, $\mathrm{tr}(\gamma^\mu\gamma^\nu\gamma^\lambda\gamma^\kappa)$ has to be proportional to products $\eta\eta$ of the metric tensor,

$$\mathrm{tr}(\gamma^\mu\gamma^\nu\gamma^\lambda\gamma^\kappa) = C[\eta^{\mu\nu}\eta^{\lambda\kappa} + \eta^{\mu\kappa}\eta^{\nu\lambda}] + D\eta^{\mu\lambda}\eta^{\nu\kappa}\,.$$

The first pair combines the neighbouring indices, e.g. $(\mu\nu)$ or $(\kappa\mu)$, in one metric tensor, the second separated ones. Since the LHS is even under parity, $\varepsilon^{\mu\nu\lambda\kappa}$ cannot appear on the RHS. (We could add $E\varepsilon^{\mu\nu\lambda\kappa}$ to the RHS, deriving below $E = 0$.) Contracting first with $\eta_{\mu\nu}$ gives

$$\mathrm{tr}(\gamma^\mu\gamma_\mu\gamma^\lambda\gamma^\kappa) = 16\eta^{\lambda\kappa} = C[4+1]\eta^{\lambda\kappa} + D\eta^{\lambda\kappa}$$

or $16 = 5C + D$. Contracting instead with $\eta_{\mu\lambda}$ gives as second relation

$$\mathrm{tr}(\gamma^\mu\gamma^\nu\gamma_\mu\gamma^\kappa) = -8\eta^{\nu\kappa} = (2C + 4D)\eta^{\nu\kappa}$$

or $C = -4 - 2D$. Combined we find $D = -4 = -C$ and

$$\mathrm{tr}[\not{a}\not{b}\not{c}\not{d}] = 4[(a \cdot b)\,(c \cdot d) - 4(a \cdot c)\,(b \cdot d) + 4(a \cdot d)\,(b \cdot c)]$$