Consider a random walk on the x-axis, the probability of going to the left and right are equal. The step-length increases with each step such that the position after N steps is:

$$X = \sum_{i=1}^{N} i\Delta_i = \Delta_1 + 2\Delta_2 + 3\Delta_3 + \cdots + N\Delta_N.$$

The random variables Δ_i can take the values $\Delta_i = \pm 1$ and they are uncorrelated: $\langle \Delta_i \Delta_j \rangle = \delta_{ij}$ where δ_{ij} is the Kronecker delta. The mean square displacement after N steps is:

Select one alternative:

- $\langle X^2 \rangle = N$
- $\langle X^2 \rangle = \sum_{i=1}^{N} i^2 = 1^2 + 2^2 + 3^2 + \cdots + N^2$
- $\langle X^2 \rangle = N^4$
- $\langle X^2 \rangle = N^2$
- $\langle X^2 \rangle = \sum_{i=1}^{N} i = 1 + 2 + 3 + \cdots + N$
- $\langle X^2 \rangle = \sqrt{N}$

Consider a particle in one dimension.
The total energy is: $E = \frac{p_x^2}{2m} + Kx$, where $K > 0$ is a constant, x the position, and p_x the momentum.
The allowed positions of the particle are on the positive x-axis: $0 \leq x \leq \infty$.
We are in the microcanonical ensemble where the total energy is constant, and the probability distribution is:
$P(p_x, x) = C\,\delta\left[\frac{p_x^2}{2m} + Kx - E\right]$
where C is a constant.
Energy conservation restricts the momentum to the interval: $-\sqrt{2mE} \leq p_x \leq \sqrt{2mE}$.
The probability distribution for p_x on this interval is:

Select one alternative:

- $P(x) = C\left(1 - \frac{p_x^2}{2mE}\right)$
- $P(p_x) = \frac{C}{K} e^{-\frac{p_x^2}{2mE}}$
- $P(p_x) = C\delta\left[\frac{p_x^2}{2m} - E\right]$
- $P(p_x) = \frac{C}{K}$
- $P(p_x) = \frac{C}{K}\frac{p_x^2}{2mE}$
- $P(p_x) = \frac{C}{p_x}$

A system has energy levels $E = n\epsilon$, where $n = 0, 1, 2, 3, \cdots, \infty$ and $\epsilon > 0$ a constant. At the energy level n there are $\Gamma_n = n^2$ number of states of the system. We are in the microcanonical ensemble and $\frac{dS}{dE} = \frac{1}{T}$. Consider that n is large and ϵ small. Express the entropy as a function of energy $S = S(E)$. What is the relation between energy and temperature in the system?

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Select one alternative:

- $E = k_B T$
- $E = \frac{\epsilon^2}{k_B T}$
- $E = 2k_B T$
- $E = k_B T\, \mathrm{e}^{-\frac{\epsilon}{k_B T}}$
- $E = \frac{(k_B T)^2}{\epsilon}$
- $E = \epsilon\, \mathrm{e}^{-\frac{\epsilon}{k_B T}}$

Consider a particle in two dimensions confined inside a recipient with area A.
The particle has the Hamiltonian: $H = \frac{p_x^2+p_y^2}{2m} + U(x,y)$. We are in the canonical ensemble. Let the potential energy be a constant: $U(x,y) = a$.
The partition function is:

Select one alternative:

- $Z = \frac{2\pi m}{h^2} k_B T A e^{-\frac{a}{k_B T}}$
- $Z = \frac{h^2}{2\pi m} (k_B T)^2 A e^{-2\frac{a}{k_B T}}$
- $Z = (\frac{2\pi m}{h^2})^{3/2} k_B T A e^{-\frac{a}{k_B T}}$
- $Z = \frac{2\pi m}{h^2} (k_B T)^2 A e^{-\frac{a}{k_B T}}$
- $Z = \frac{2\pi m}{h^2} k_B T A^2 e^{\frac{a}{k_B T}}$
- $Z = \frac{2\pi m}{h^2} \frac{A}{k_B T} e^{\frac{a}{k_B T}}$

A system has the density of states: $\rho(E) = C\mathrm{e}^{-E/E_0}$ with $E \geq 0$
and E_0 and C are positive constants. We are in the canonical ensemble
and the partition function is $Z = \int_0^\infty dE\,\rho(E)e^{-\beta E}$.
The mean energy is:

Select one alternative:

- $\langle E \rangle = E_0$
- $\langle E \rangle = 0$
- $\langle E \rangle = \frac{(k_B T)^2}{E_0}$
- $\langle E \rangle = \frac{E_0\,k_B T}{E_0 + k_B T}$
- $\langle E \rangle = k_B T$
- $\langle E \rangle = \frac{E_0^2}{k_B T}$

Consider a single particle inside a container of volume V.
The container is divided into two parts of equal volume $\frac{V}{2}$,
in one side the potential energy of the particle is $U=\epsilon$,
in the other part the potential energy is $U=0$. Calculate
the average potential energy of the particle $\langle U \rangle$ in the
canonical ensemble. At any given instant what is the probability
P of finding the particle in the part of the container with
energy $U=\epsilon$?

Select one alternative:

- $P=\frac{1}{2}-\frac{\epsilon}{k_B T}$
- $P=\frac{1}{(e^{\beta\epsilon}+1)^2}$
- $P=\frac{1}{e^{\beta\epsilon}-1}$
- $P=\frac{1}{2}-\left(\frac{\epsilon}{k_B T}\right)^2$
- $P=\frac{1}{e^{\beta\epsilon}+1}$
- $P=\frac{1}{2}$

Consider a particle in one dimension with Hamiltonian $H = \frac{p_x^2}{2m} + \frac{k_B T}{L} x$ and the constraint $0 \leq x \leq \infty$. We are in the canonical ensemble. What is the probability p that the the position of the particle is $x \geq L$?

Select one alternative:

- $p = \frac{1}{4}$
- $p = \frac{1}{2}$
- $p = \mathrm{e}^{-1}$
- $p = 1 - \mathrm{e}^{-2}$
- $p = 1 - \mathrm{e}^{-3}$
- $p = 1 - \mathrm{e}^{-1}$

A particle with Hamiltonian $H = \frac{p_x^2}{2m}$ is moving freely (no external forces) in one dimension. The phase space is (x, p_x). Consider a rectangle in phase space at time $t = 0$, defined by the four corners (x, p_x), $(x + \Delta x, p_x)$, $(x, p_x + \Delta p_x)$, $(x + \Delta x, p_x + \Delta p_x)$.
Consider the area defined by all the points inside this rectangle.
What happens to this area as time evolves?

Select one alternative:

- The area in phase space is just locally preserved, therefore the area changes since the rectangle has finite size.
- The area in phase space is not preserved. The circumference is preserved.
- The rectangle is simply translated and does not change shape.
- The area is conserved, and the circumference of the area goes toward a constant.
- The area is conserved and the circumference goes to zero.
- The area is conserved, but the rectangle is deformed such that the circumference of the area diverges.

A classical non-relativistic particle in two dimensions is confined inside a circle of radius R. The particle is subject to a potential $U(r) = -\epsilon$ when $r < R_0$, and $U(r) = 0$ when $R_0 < r < R$.
r is the distance from the origin (center of circle). The mass of the particle is m.
The canonical partition function of the particle is:

Select one alternative:

- $Z = \frac{2\pi m}{\beta h^2} \pi R^2 \, \mathrm{e}^{\beta\epsilon}$
- $Z = \left(\frac{2\pi m}{\beta h^2}\right)^{3/2} \pi (R^2 - R_0^2) \, \mathrm{e}^{\beta\epsilon}$
- $Z = \frac{2\pi m}{\beta h^2} \pi (R^2 - R_0^2) \, \mathrm{e}^{\beta\epsilon}$
- $Z = \frac{2\pi m}{\beta h^2} \left[\pi R^2 + \pi R_0^2 \, \mathrm{e}^{\beta\epsilon}\right]$
- $Z = \frac{2\pi m}{\beta h^2} \left[\pi (R^2 - R_0^2) \, \mathrm{e}^{-\beta\epsilon} + \pi R_0^2 \, \mathrm{e}^{\beta\epsilon}\right]$
- $Z = \frac{2\pi m}{\beta h^2} \left[\pi (R^2 - R_0^2) + \pi R_0^2 \, \mathrm{e}^{\beta\epsilon}\right]$

An ideal classical two-dimensional gas of N particles is confined inside an area A. All particles have the same mass m. The canonical partition function is:

Select one alternative:

- $Z_N = \frac{1}{N! h^{2N}} (2\pi m k_B T)^N A^N$
- $Z_N = \frac{1}{N! h^{2N}} (2\pi m k_B T)^{-2N} A^{2N}$
- $Z_N = \frac{1}{N! h^{2N}} (\frac{2\pi m}{k_B T})^N A^N$
- $Z_N = \frac{1}{N! h^{2N}} (2\pi m k_B T)^{N/2} A^N$
- $Z_N = \frac{1}{N! h^{2N}} (\frac{2\pi m}{k_B T})^{2N} A^{2N}$
- $Z_N = \frac{1}{N! h^{2N}} (2\pi m k_B T)^N A^{N/2}$

A system of N classical distinguishable particles can occupy two energy levels, one ground state and one excited state. How many different configurations are there with 2 particles in the ground state?

Select one alternative:

- $\frac{N!}{2!}$
- $N^2 - N$
- $\frac{N(N-1)}{2}$
- $\frac{N}{2}$
- N
- $\frac{N^2}{2}$

In classical statistical mechanics we divide by a factor $N!$ when we count the states of N particles (in a gas for example). Which one of these statements is correct?

Select one alternative:

- The factor N! is only used in the micro-canonical ensemble.
- The factor N! is only important at low temperatures.
- The factor N! is not important when calculating the grand partition function of a system.
- The factor N! makes entropy finite both in the canonical and micro-canonical ensemble.
- The factor N! makes entropy an extensive quantity both in the canonical and micro-canonical ensemble.
- The factor N! is only used in the canonical ensemble.

Which one of these statements is correct for the Debye model for the heat capacity of solids?

Select one alternative:

- The heat capacity in the low temperature limit comes mostly from the long wavelength elastic deformations (phonons) in the solid.
- The heat capacity in the low temperature limit is the sum of the heat capacity of individual atoms, interactions between the atoms can be neglected.
- The model is only valid in the low temperature limit.
- The heat capacity in the high temperature limit comes mostly from the long wavelength elastic deformations (phonons) in the solid.
- The model is only valid in the high temperature limit.
- The density of states is a delta-function in frequency.

A system with N particles has the canonical partition function $Z = \mathrm{e}^{-\beta\epsilon N}$.
The variance of the energy $\Delta E^2 = \langle E^2 \rangle - \langle E \rangle^2$ is:

Select one alternative:

- $\Delta E^2 = \epsilon^2$
- $\Delta E^2 = \frac{\epsilon^2}{N}$
- $\Delta E^2 = \sqrt{N}\epsilon^2$
- $\Delta E^2 = N\epsilon^2$
- $\Delta E^2 = N^2\epsilon^2$
- $\Delta E^2 = 0$

Consider a system of three spins with Hamiltonian: $H = -Js_1s_2 - 2Js_2s_3$, where J is a postive constant, and $s_i = \pm 1$. We are in the canonical ensemble. In the high temperature limit $k_BT \gg J$ the free energy $F = \langle E \rangle - TS$ is:

Select one alternative:

- $F \approx -3J$
- $F \approx -\ln(2)k_BT$
- $F \approx -3\ln(2)k_BT$
- $F \approx \frac{3}{2}k_BT$
- $F \approx -\frac{(k_BT)^2}{J}$
- $F \approx 3k_BT$

Consider a system of three spins with Hamiltonian: $H = -Js_1s_2 + Js_2s_3$, where J is a postive constant, and $s_i = \pm 1$. We are in the canonical ensemble. When $T \to 0$ the mean energy $\langle E \rangle$ and heat capacity C become:

Select one alternative:

- $\langle E \rangle = 0$ and $C = k_B$
- $\langle E \rangle = -2J$ and $C = k_B$
- $\langle E \rangle = -2J$ and $C = 0$
- $\langle E \rangle = 0$ and $C = 0$
- $\langle E \rangle = -2J$ and $C = 2k_B$
- $\langle E \rangle = -J$ and $C = 2k_B$

A system is described by the Hamiltonian $H = J(s_1 s_2 + s_2 s_3 + s_3 s_1)$ where $J > 0$ and the spins take the values $s_i = \pm 1$. When $T \to 0$ the mean energy and entropy become:

Select one alternative:

- $\langle E \rangle = -2J$ and $S > 0$
- $\langle E \rangle = -3J$ and $S > 0$
- $\langle E \rangle = -2J$ and $S = 0$
- $\langle E \rangle = -J$ and $S = 0$
- $\langle E \rangle = -J$ and $S > 0$
- $\langle E \rangle = -3J$ and $S = 0$

A system of spins $s_i = \pm 1$ is such that half of the spins align and the other half anti-align when subject to an external field h. The Hamiltonian is
$H = -h(s_1 + s_2 + \cdots + s_N) + h(s_{N+1} + s_{N+2} + \cdots + s_{2N})$
with $h > 0$. We are in the canonical ensemble. The free energy of the system is:

Select one alternative:

- $F = -2N\frac{(k_BT)^2}{h}$
- $F = -\frac{N}{\beta}\ln\left(e^{2\beta h} + e^{-2\beta h}\right)$
- $F = -\frac{2N}{\beta}\ln\left(e^{\beta h} + e^{-\beta h}\right)$
- $F = -2N\frac{h^2}{k_BT}$
- $F = -\frac{2N}{\beta}\ln\left(1 + e^{2\beta h}\right)$
- $F = -\frac{2N}{\beta}\ln\left(1 + e^{-2\beta h}\right)$

A system has the Hamiltonian $H = -J\phi_1^2\phi_2$, where $J > 0$ is a constant and the variables ϕ_i can take two values $\phi_i = 1$ or $\phi_i = -1$. We are in the canonical ensemble. The average value of $\phi_1^2\phi_2$ is:

Select one alternative:

- $\langle \phi_1^2\phi_2 \rangle = \mathrm{e}^{-\beta J}$
- $\langle \phi_1^2\phi_2 \rangle = \frac{\mathrm{e}^{\beta J} - \mathrm{e}^{-\beta J}}{\mathrm{e}^{\beta J} + \mathrm{e}^{-\beta J}}$
- $\langle \phi_1^2\phi_2 \rangle = 1 - \mathrm{e}^{-2\beta J}$
- $\langle \phi_1^2\phi_2 \rangle = 1 - \frac{1}{1+(\beta J)^2}$
- $\langle \phi_1^2\phi_2 \rangle = \frac{\mathrm{e}^{2\beta J} - \mathrm{e}^{-2\beta J}}{\mathrm{e}^{2\beta J} + \mathrm{e}^{-2\beta J}}$
- $\langle \phi_1^2\phi_2 \rangle = 1 - \mathrm{e}^{-\beta J}$

A paramagnet is given by the Hamiltonian:

$$H = \sum_{i=1}^{N}(-hs_i + \epsilon s_i^2) = -h(s_1 + s_2 + \cdots + s_N) + \epsilon(s_1^2 + s_2^2 + \cdots + s_N^2)$$

where $s_i = \pm 1$, and h and ϵ are positive constants, and $\epsilon \ll h$. We are in the canonical ensemble. When $T = 0$ the entropy is :

Select one alternative:

- $S = k_B \ln 2$
- $S = Nk_B$
- $S = 0$
- $S = Nk_B \frac{\epsilon}{h}$
- $S = Nk_B \ln 2$
- $S = N\frac{k_B}{2}$

N non-interacting vector spins in the x-y plane are oriented by an external field $\vec{h}$. The Hamiltonian is:

$H = -\vec{h} \cdot (\vec{s}_1 + \vec{s}_2 + \cdots + \overrightarrow{s_N})$

where $\vec{s}_i = (\cos\theta_i, \sin\theta_i)$ with $-\pi \leq \theta_i \leq \pi$. The field $\vec{h} = (h, 0) = h\vec{e}_x$ is oriented along the x-axis. The mean energy in the low temperature limit $k_B T \ll h$ is:

Select one alternative:

- $\langle E \rangle \approx -Nh + N\frac{(k_B T)^2}{h}$
- $\langle E \rangle \approx -Nh - Nk_B T$
- $\langle E \rangle \approx -Nh + N\frac{(k_B T)^3}{h^2}$
- $\langle E \rangle \approx \frac{N}{2} k_B T$
- $\langle E \rangle \approx -Nh + \frac{N}{2} k_B T$
- $\langle E \rangle \approx N\frac{(k_B T)^3}{h^2}$

Consider a system that has free energy $F = k_B TN - k_B TN \ln(N)$ where N is the number of particles in the system. The volume is constant. The system is connected to a reservoir of N_r particles with free energy $F_r = N_r\mu$, where $\mu < 0$ is a constant chemical potential. Assume that N is macroscopically large and that we always have $N_r \gg N$. What is the number of particles in the system in thermal equilibrium?

Select one alternative:

- $N = \exp\left(-\frac{\mu}{k_B T}\right)$
- $N = -\frac{\mu}{k_B T}$
- $N = (\frac{\mu}{k_B T})^2$
- $N = \exp\left(-\frac{\mu}{k_B T}\right) + \exp\left(-\frac{2\mu}{k_B T}\right)$
- $N = -\frac{\mu}{k_B T}\exp\left(-\frac{\mu}{k_B T}\right)$
- $N = \frac{2}{\exp\left(-\frac{\mu}{k_B T}\right)-1}$

Consider a classical two-dimensional ideal gas of N molecules in a container of constant area A. The chemical potential of the molecules in the gas is:
$\mu = k_B T \ln(N\lambda^2/A)$, where λ is the thermal de Broglie length.
The molecules can adsorb onto the walls of the container. The gas molecules adsorbed on the walls have chemical potential:
$\mu_s = -\epsilon + k_B T \ln(N_s\lambda^2/A_s)$, where $\epsilon > 0$, N_s is the number of adsorbed molecules, and A_s the surface area in which the molecules adsorb. The total number of molecules in the container is $N_t = N + N_s$. In equilibrium the mean number of molecules on the surface can be expressed as:

Select one alternative:

- $N_s = N\frac{A_s}{A}$
- $N_s = N\exp\left(\frac{\epsilon}{k_BT}\right)$
- $N_s = N\frac{A_s^2}{A^2}\exp\left(\frac{\epsilon}{k_BT}\right)$
- $N_s = \frac{N}{1+\frac{A_s}{A}\exp\left(-2\frac{\epsilon}{k_BT}\right)}$
- $N_s = \frac{N}{1+\exp\left(-\frac{\epsilon}{k_BT}\right)}$
- $N_s = N\frac{A_s}{A}\exp\left(\frac{\epsilon}{k_BT}\right)$

A system has a grand canonical partition function:

$$\Theta = \exp\left[\beta\mu + (\beta\mu)^2\right]$$

What is the average energy $\langle E \rangle$ of the system?

Select one alternative:

- $\langle E \rangle = \beta\mu^2$
- $\langle E \rangle = \mu$
- $\langle E \rangle = \frac{1}{\beta} + 3\mu$
- $\langle E \rangle = -\mu - \beta\mu^2$
- $\langle E \rangle = 0$
- $\langle E \rangle = \frac{1}{\beta}$

A system has grand partition function:
$\Theta = \exp(M\beta\mu - \beta\epsilon)$
where $\epsilon > 0$ is a constant, and $M \gg 1$ a constant number. μ is the chemical potential and $\beta = \frac{1}{k_B T}$. The fluctuations of the particle number $\Delta N^2 = \langle N^2 \rangle - \langle N \rangle^2$ in the system is:

Select one alternative:

- $\Delta N^2 = \infty$
- $\Delta N^2 = 0$
- $\Delta N^2 = M \exp\left(\frac{\epsilon}{\mu}\right)$
- $\Delta N^2 = M \frac{\epsilon}{\mu}$
- $\Delta N^2 = M$
- $\Delta N^2 = M^2$

A system has canonical partition function Z_N, where N is the number of particles. The system cannot have more than N_c particles.
When $N \leq N_c$ the partition function is $Z_N = \mathrm{e}^{\beta\epsilon N}$, and when $N > N_c$ the partition function is $Z_N = 0$. $\epsilon > 0$ is a constant. The system is connected to a particle reservoir with chemical potential $\mu = 0$.
When $\beta\epsilon \gg 1$ the grand partition function is:

Select one alternative:

- $\Theta \approx N_c \exp\left(N_c\, \beta\epsilon\right)$
- $\Theta \approx \exp\left(\,\beta\epsilon\right)$
- $\Theta \approx 1$
- $\Theta \approx N_c \exp\left(\,\beta\epsilon\right)$
- $\Theta \approx \exp\left(\exp\left(N_c\, \beta\epsilon\right)\right)$
- $\Theta \approx \exp\left(N_c\, \beta\epsilon\right)$

A system has canonical partition function $Z_N = KV^{N/3}(k_BT)^{N/5}$, where K is a constant, N the number of particles and V the volume. The pressure p and heat capacity C of the system are:

Select one alternative:

- $p = 0$ and $C = \frac{N}{3}k_B$
- $p = \frac{N}{V}k_BT$ and $C = \frac{3N}{2}k_B$
- $p = \frac{N}{3V}k_BT$ and $C = \frac{N}{5}k_B$
- $p = \frac{N}{3V}k_BT$ and $C = 0$
- $p = \frac{1}{3V}k_BT$ and $C = \frac{1}{5}k_B$
- $p = \frac{3N}{5V}k_BT$ and $C = \frac{5N}{3}k_B$

A box of volume V contains an ideal classical gas of N molecules, with chemical potential $\mu_g = k_B T \ln(N\lambda^3/V)$, λ is the thermal de Broglie length. We introduce two spherical objects (beads) inside the box, and the molecules can adsorb on these objects. The objects are far apart (not touching). The chemical potential of adsorption on sphere 1 is:

$$\mu_1 = -\epsilon_1 + k_B T \ln(c N_1)$$

and similarly for sphere 2:

$$\mu_2 = -\epsilon_2 + k_B T \ln(c N_2)$$

where N_1 and N_2 are the number of adsorbed molecules on sphere 1 and 2, and ϵ_1,ϵ_2 c are constants. We always have $N_1 \ll N$ and $N_2 \ll N$. In thermal equilibrium the ratio $\frac{N_1}{N_2}$ is:

Select one alternative:

- $\frac{N_1}{N_2} = \exp\left[-\frac{\epsilon_1}{k_B T}\right] + \exp\left[-\frac{\epsilon_2}{k_B T}\right]$
- $\frac{N_1}{N_2} = \exp\left[\frac{(\epsilon_1 - \epsilon_2)}{k_B T}\right]$
- $\frac{N_1}{N_2} = \frac{\epsilon_1}{\epsilon_2}\exp\left[-\frac{\epsilon_1}{k_B T}\right] + \frac{\epsilon_2}{\epsilon_1}\exp\left[-\frac{\epsilon_2}{k_B T}\right]$
- $\frac{N_1}{N_2} = \frac{\epsilon_1}{\epsilon_2}\exp\left[\frac{(\epsilon_1 - \epsilon_2)}{k_B T}\right]$
- $\frac{N_1}{N_2} = 1$
- $\frac{N_1}{N_2} = \frac{\epsilon_1}{\epsilon_2}$

Consider the 1D Ising model with an external magnetic field:

$$H = -J\sum_{i=1}^{N-1} s_i s_{i+1} - \mu B \sum_{i=1}^{N} s_i$$

where μ is the magnetic moment. We are in the canonical ensemble.
The heat capacity is $C = \frac{\partial}{\partial T}\langle H \rangle$. The heat capacity at zero and infinite temperature is denoted by $C_0 = C(T=0)$ and $C_\infty = C(T \to \infty)$.
Which one of these statements is correct?

Select one alternative:

- $C_0 = 0$ and $C_\infty = Nk_B$
- $C_0 = 0$ and $C_\infty = k_B \frac{\mu B}{J}$
- $C_0 = Nk_B$ and $C_\infty = 0$
- $C_0 = 0$ and $C_\infty = 0$
- $C_0 = 0$ and $C_\infty = \infty$
- $C_0 = Nk_B$ and $C_\infty = Nk_B$

A polymer in two dimensions has 5 monomers, numbered $n = 0, 1, 2, 3, 4$. The position of monomer n is $\vec{R}_n = \sum_{i=1}^{n} \vec{t}_i$, where $\vec{t}_i$ are the bond vectors.

The angles θ_i between two consecutive bond vectors ($\vec{t}_i \cdot \vec{t}_{i+1} = \cos(\theta_i)$) can only take three values: $\theta_i = 0, \alpha, -\alpha$

The deformation energy of the polymer is $H = -\epsilon(\cos\theta_1 + \cos\theta_2 + \cos\theta_3)$.
The canonical partition function $Z = \sum_{\theta_1,\theta_2,\theta_3} \exp(-\beta H)$ of the polymer is:

Select one alternative:

- $Z = \exp(\beta\epsilon) + 2\exp[\beta\epsilon\cos(\alpha)]$
- $Z = Z_1^3$ where $Z_1 = 1 + 2\exp[\beta\epsilon]$
- $Z = Z_1^4$ where $Z_1 = 1 + 2\exp[\beta\epsilon\cos(\alpha)]$
- $Z = Z_1^3$ where $Z_1 = 1 + 2\exp[\beta\epsilon\cos(\alpha)]$
- $Z = Z_1^2$ where $Z_1 = 1 + 2\exp[\beta\epsilon]$
- $Z = Z_1^3$ where $Z_1 = \exp(\beta\epsilon) + 2\exp[\beta\epsilon\cos(\alpha)]$

In the Debye model for heat capacity of solids it is assumed that the density of states is quadratic $g(\omega) \propto \omega^2$ up to the cut-off frequency ω_D. This gives a heat capacity at low temperature that is proportional to temperature to the power three: $C \propto T^3$. What would be the low temperature behaviour if we instead assumed that the density of states is linear: $g(\omega) \propto \omega$ up to the cut-off frequency.

Select one alternative:

- $C(T) \sim T^4$
- $C(T) \sim T^3$
- $C(T) \sim T^{3/2}$
- $C(T) \sim T$
- $C(T) \sim T^2$
- $C(T) \sim T^{5/2}$

Consider a system with Hamiltonian $H_N = -h(s_1 + s_2 + \cdots + s_N)$,
where $h > 0$, $s_i = \pm 1$ and N is the number of particles in the system.
The canonical partition function is given by: $Z_N = \sum_{\{s_i\}} \mathrm{e}^{-\beta H_N}$.

We are in the grand canonical ensemble and the chemical potential is μ. The grand partition function is a convergent series when $N \to \infty$ provided $\mu < 0$ is sufficiently small, in which case Θ becomes:

Select one alternative:

- $\Theta = \frac{1}{1+\mathrm{e}^{2\beta\mu}(\mathrm{e}^{2\beta h}+\mathrm{e}^{-2\beta h})}$
- $\Theta = \frac{1}{\mathrm{e}^{\beta\mu}+\mathrm{e}^{\beta h}+\mathrm{e}^{-\beta h}}$
- $\Theta = \frac{1}{1-\mathrm{e}^{\beta\mu}(\mathrm{e}^{\beta h}+\mathrm{e}^{-\beta h})}$
- $\Theta = \frac{1}{1-\mathrm{e}^{\beta\mu}}$
- $\Theta = \frac{1}{1-\mathrm{e}^{\beta h}-\mathrm{e}^{-\beta h}}$
- $\Theta = \frac{1}{1-\mathrm{e}^{\beta h}-\mathrm{e}^{-\beta h}-\mathrm{e}^{\beta\mu}}$

The grand partition function of a classical ideal gas is:

$$\Theta = \exp\left[\frac{V}{\lambda^3}\exp(\beta\mu)\right]$$

where λ is the thermal de Broglie length. The density of the gas is $\rho = \frac{\langle N\rangle}{V}$. What happens to the density if we keep μ constant and let $T \to \infty$?

Select one alternative:

- Approaches zero following high temperature behaviour $\rho \propto 1/\sqrt{T}$
- Approaches a constant density $\rho = \text{constant}$
- Approaches zero following high temperature behaviour $\rho \propto 1/T^2$
- Approaches zero following high temperature behaviour $\rho \propto 1/T^4$
- Diverges following the high temperature behaviour $\rho \propto T^{3/2}$
- Diverges following the high temperature behaviour $\rho \propto T^3$

Consider a particle living on the z-axis, and subject to a gravitational field. The Hamiltonian is: $H = \frac{p_z^2}{2m} + mgz$. The particle is confined to the interval $0 \leq z \leq L$. We can think of this as a particle in a one-dimensional box. We are in the canonical ensemble. The pressure at $z = L$ is defined as $P = -\frac{\partial F}{\partial L}$ where $F = -\frac{1}{\beta} \ln Z$ is the free energy of the particle. Calculate F and find P. Which of these is the correct expression for the pressure P?

Select one alternative:

- $P = \frac{mg}{L}$
- $P = \frac{k_B T}{L^2}$
- $P = \frac{mg}{e^{\beta mgL} - 1}$
- $P = \frac{k_B T}{2L}$
- $P = \frac{mg}{L}$
- $P = \frac{k_B T}{e^{\beta mgL} + 1}$

A system has the possible energy levels $E = n\epsilon$ where $n = 0, 1, 2, 3, \cdots, N$, and $\epsilon > 0$ a constant. For a given energy level n there are $\Gamma_n = \frac{N!}{(N-n)!\, n!}$ different configurations of the system. We are in the microcanonical ensemble. Calculate the entropy $S = k_B \ln \Gamma_n$ using Stirling's formula, and assuming that N, $N-n$ and n are all large numbers.
The relation $\frac{dS}{dE} = \frac{1}{T}$ implies:

Select one alternative:

- $\frac{n}{N-n} = \mathrm{e}^{-\frac{\epsilon}{k_B T}}$
- $\frac{n}{N-n} = \frac{k_B T}{\epsilon}$
- $\frac{n}{2N} = \mathrm{e}^{-\frac{2\epsilon}{k_B T}}$
- $\frac{n}{N} = \frac{\epsilon}{k_B T}$
- $\frac{n}{N} = \frac{k_B T}{\epsilon}$
- $n = \mathrm{e}^{\frac{\epsilon}{k_B T}}$

Consider a system with three energy levels $\epsilon_0 = 0$, $\epsilon_1 = \epsilon$, and $\epsilon_2 = 2\epsilon$. The system is populated with $N = 2$ bosons following Bose-Einstein statistics. The canonical partition function is:

Select one alternative:

- $Z = 1 + \mathrm{e}^{-3\beta\epsilon} + \mathrm{e}^{-6\beta\epsilon}$
- $Z = 1 + \mathrm{e}^{-\beta\epsilon} + 2\mathrm{e}^{-2\beta\epsilon} + \mathrm{e}^{-3\beta\epsilon} + \mathrm{e}^{-4\beta\epsilon}$
- $Z = \mathrm{e}^{-\beta\epsilon} + \mathrm{e}^{-2\beta\epsilon} + \mathrm{e}^{-3\beta\epsilon} + \mathrm{e}^{-4\beta\epsilon}$
- $Z = 1 + \mathrm{e}^{-\beta\epsilon} + \mathrm{e}^{-2\beta\epsilon} + \mathrm{e}^{-3\beta\epsilon}$
- $Z = 1 + \mathrm{e}^{-4\beta\epsilon}$
- $Z = 1 + \mathrm{e}^{-\beta\epsilon} + 2\mathrm{e}^{-2\beta\epsilon} + 4\mathrm{e}^{-3\beta\epsilon} + 8\mathrm{e}^{-4\beta\epsilon}$

A system with three energy levels $\epsilon_1 = 0$,$\epsilon_1 = \epsilon$, and $\epsilon_1 = 2\epsilon$ is populated by $N = 2$ indistinguishable fermions obeying Fermi-Dirac statistics. The canonical partition function is:

Select one alternative:

- $Z = 1 + \mathrm{e}^{3\beta\epsilon}$
- $Z = \mathrm{e}^{\beta\epsilon} + \mathrm{e}^{3\beta\epsilon}$
- $Z = \mathrm{e}^{-\beta\epsilon} + \mathrm{e}^{-2\beta\epsilon}$
- $Z = \mathrm{e}^{-\beta\epsilon} + \mathrm{e}^{-2\beta\epsilon} + \mathrm{e}^{-3\beta\epsilon}$
- $Z = 1 + \mathrm{e}^{-\beta\epsilon} + \mathrm{e}^{-2\beta\epsilon}$
- $Z = \mathrm{e}^{-2\beta\epsilon} + \mathrm{e}^{-4\beta\epsilon}$

Consider a system of bosons in one dimension inside a harmonic potential.
The energy is $\epsilon = (n + \frac{1}{2})\hbar\omega$, where $n = 0, 1, 2, \cdots, \infty$. We are in the grand canonical ensemble.
Find the density of states $g(\epsilon)$. In the continuum limit the average number of particles in the excited states (not ground state) is:

Select one alternative:

- $\langle N_{ex} \rangle = \frac{1}{(\hbar\omega)^2} \int_0^\infty \frac{\mathrm{d}\epsilon\,\epsilon}{\mathrm{e}^{\beta(\epsilon-\mu)}-1}$
- $\langle N_{ex} \rangle = \frac{1}{(\hbar\omega)^3} \int_0^\infty \frac{\mathrm{d}\epsilon\,\epsilon}{\mathrm{e}^{\beta(\epsilon-\mu)}-1}$
- $\langle N_{ex} \rangle = \frac{1}{\hbar\omega} \int_0^\infty \frac{\mathrm{d}\epsilon}{\mathrm{e}^{\beta(\epsilon-\mu)}-1}$
- $\langle N_{ex} \rangle = \int_0^\infty \frac{\mathrm{d}\epsilon}{\mathrm{e}^{\beta(\epsilon-\mu)}-1}$
- $\langle N_{ex} \rangle = \int_0^\infty \frac{\mathrm{d}\epsilon\,\hbar\omega^2}{\mathrm{e}^{\beta(\epsilon-\mu)}-1}$
- $\langle N_{ex} \rangle = \frac{1}{\hbar\omega} \int_0^\infty \frac{\mathrm{d}\epsilon\,\epsilon^2}{\mathrm{e}^{\beta(\epsilon-\mu)}-1}$

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From the expression for the energy we get $\mathrm{d}\epsilon = \hbar\omega \mathrm{d}n$ which implies that the density of states is $g(\omega) = \frac{1}{\hbar\omega}$.

The number of excited states is therefore

$$\langle N_{ex} =\rangle \frac{1}{\hbar\omega} \int_0^\infty \frac{\mathrm{d}\epsilon}{\mathrm{e}^{\beta(\mu-\epsilon)} - 1}$$

A system has three energy levels, a ground state $\epsilon_1 = 0$ and two excited states with the same energy $\epsilon_2 = \epsilon$, $\epsilon_3 = \epsilon$. The system is populated with bosons obeying Bose-Einstein statistics. We are in the grand canonical ensemble. The average number of particles in the system is $\langle N \rangle$. On average half of the particles are in the ground state, what is the temperature?

Select one alternative:

- $k_B T = \frac{\epsilon}{\ln\left(\frac{N+4}{N+2}\right)}$
- $k_B T = \frac{\epsilon}{\ln(2)}$
- $k_B T = 2\epsilon$
- $k_B T = \epsilon$
- $k_B T = \frac{\epsilon}{2N}$
- $k_B T = \frac{\epsilon}{\ln(N)}$

Consider a single gas particle with kinetic energy $E = \frac{p_x^2+p_y^2+p_z^2}{2m}$ confined inside a volume V. We are in the canonical ensemble. What is the fluctuations in energy of the particle $\Delta E^2 = \langle E^2 \rangle - \langle E \rangle^2$?

Select one alternative:

- $\Delta E^2 = \frac{1}{4}(k_B T)^2$
- $\Delta E^2 = (k_B T)^2$
- $\Delta E^2 = 2(k_B T)^2$
- $\Delta E^2 = \frac{3}{5}(k_B T)$
- $\Delta E^2 = 4(k_B T)^2$
- $\Delta E^2 = \frac{3}{2}(k_B T)^2$