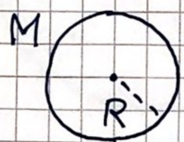
22.09.21

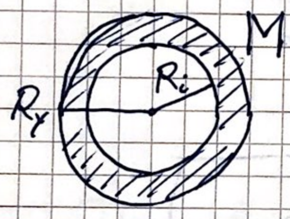
Beregning av treghetsmoment

Eks 1: Tynn ring / Hul sylinder



$$I_0 = \int \rho^2 \, dm = R^2 \int dm = \underline{MR^2}$$

Eks 2: Ring med en viss tykkelse



"Triks": Tynn ring med radius ρ (mellom R_i og R_y her) og tykkelse $d\rho$ har areal $dA = 2\pi\rho \cdot d\rho$ (omkrets · tykkelse)

$$\Rightarrow \frac{dm}{M} = \frac{2\pi\rho \, d\rho}{\pi(R_y^2 - R_i^2)}$$

$$\Rightarrow I_0 = \int \rho^2 \, dm = \int_{R_i}^{R_y} \rho^2 \cdot M \cdot \frac{2\rho \, d\rho}{R_y^2 - R_i^2}$$

$$= \frac{2M}{R_y^2 - R_i^2} \cdot \Big|_{R_i}^{R_y} \frac{1}{4}\rho^4 = \underline{\underline{\frac{1}{2} M (R_y^2 + R_i^2)}}$$

$$\left(\text{siden } R_y^4 - R_i^2 = (R_y^2 + R_i^2)(R_y^2 - R_i^2) \right)$$

Ringen på laben:

$R_y = 25 \text{ mm}$, $R_i = 22 \text{ mm}$

$$\Rightarrow I_0 \approx 0.89 \, MR^2 \qquad (R = R_y)$$

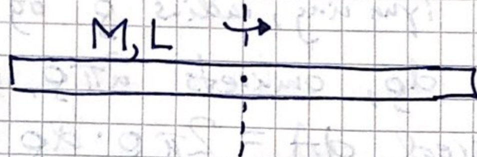
Eks 3 : Kompakt skive

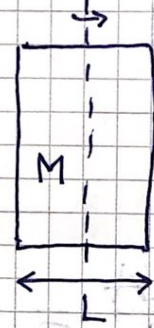
$= $ Eks 2 med $R_i \to 0$

$$\Rightarrow \underline{\underline{I_0 = \frac{1}{2} M R^2}}$$

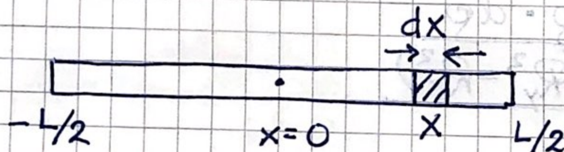
(og Eks 1 er Eks 2 med $R_i \to R_y \Rightarrow I_0 = M R^2$)

Eks 4 : Tynn stang (evt tynn plate, f.eks dør)





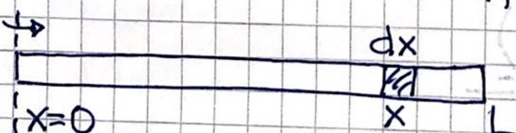
svingdør !

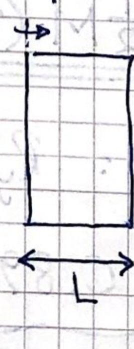


$\rho \to x$; $dm / M = dx / L$

$$\Rightarrow I_0 = \int_{-L/2}^{L/2} x^2 \cdot \frac{M \cdot dx}{L} = \underline{\underline{\frac{1}{12} M L^2}}$$

Eks 5 : Som Eks 4, men mhp akse ved enden





vanlig dør

$$I = \int_0^L x^2 M\, dx / L = \underline{\underline{\frac{1}{3} M L^2}}$$

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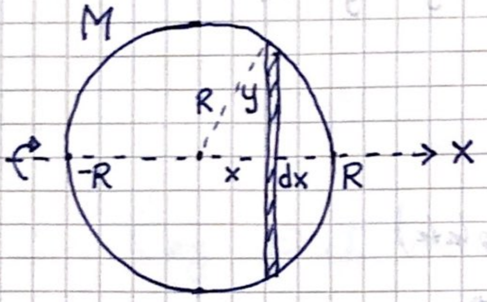
Eks 6: Kompakt kule



= skiver med tykkelse dx,
radius $y(x) = \sqrt{R^2 - x^2}$,
treghetsmoment $dI_o = \frac{1}{2} \cdot dm \cdot y^2$;
$-R \leq x \leq R$

$$\frac{dm}{M} = \frac{dV}{V} = \frac{\pi y^2 \cdot dx}{\frac{4}{3}\pi R^3} = \frac{3}{4} \cdot \frac{R^2 - x^2}{R^3} \cdot dx$$

$$\Rightarrow I_o = \int dI_o = \int_{-R}^{R} \frac{1}{2} \cdot M \cdot \frac{3}{4} \cdot \frac{R^2 - x^2}{R^3} \cdot dx \cdot (R^2 - x^2)$$

$$= \frac{3M}{8R^3} \int_{-R}^{R} (R^4 - 2R^2x^2 + x^4)\, dx$$

$$= \frac{3M}{8R^3} \cdot \Big|_{-R}^{R} \left(R^4 x - \frac{2}{3}R^2x^3 + \frac{1}{5}x^5\right)$$

$$= \frac{3M}{8R^3} \cdot R^5 \cdot \left(2 - \frac{4}{3} + \frac{2}{5}\right)$$

$$= MR^2 \cdot \frac{3}{8} \cdot \frac{30 - 20 + 6}{15}$$

$$= \underline{\underline{\frac{2}{5} MR^2}}$$

Eks 7: Tynt kuleskall = ringer med bredde dx, radius $\sqrt{R^2 - x^2}$, treghetsmoment $dI_o = dm \cdot y^2$, og $dm/M = dA/A = 2\pi y \cdot dx / 4\pi R^2 \Rightarrow \underline{\underline{I_o = \frac{2}{3} MR^2}}$

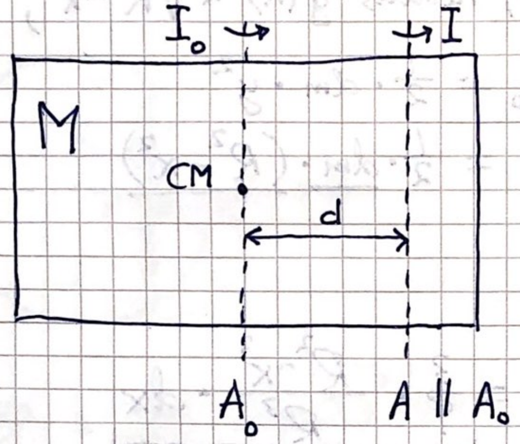
$\boxed{\text{Hit 22.09.21 Uke 38}}$

(33)

(Uke 39!)

27.09.21

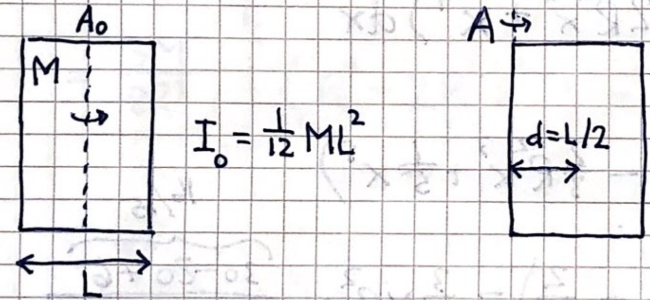
Steiners sats (Parallellakseteoremet)



$$\boxed{I = I_0 + Md^2}$$

som gjelder generelt når aksene A_0 og A er parallelle

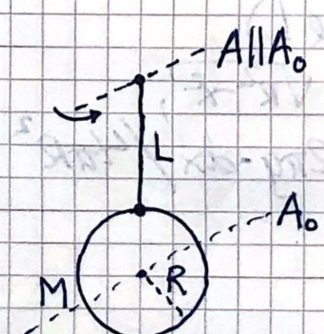
Eks 1: Svingdør vs vanlig dør



$I_0 = \frac{1}{12}ML^2$

$$I = I_0 + Md^2$$
$$= \frac{1}{12}ML^2 + M \cdot (L/2)^2$$
$$= \underline{\underline{\frac{1}{3}ML^2}} \quad \text{(som s. 31)}$$

Eks 2: Julekule i tråd



$I_0 = \frac{2}{3}MR^2$ (kuleskall)

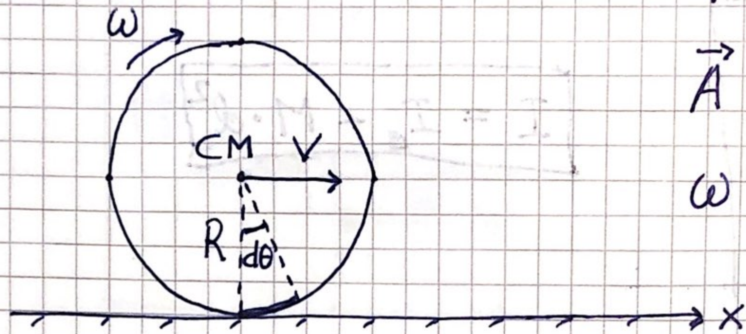
$$d = R + L$$

$$\Rightarrow$$

$$\underline{\underline{I = \frac{2}{3}MR^2 + M \cdot (R+L)^2}}$$

Fysisk pendel; svingetida (perioden) T er prop. med $\sqrt{I}$

Rulling



$$\vec{V} = \frac{d\vec{R}_{CM}}{dt} = \dot{\vec{R}}_{CM}$$

$$\vec{A} = \ddot{\vec{R}}_{CM} = \dot{\vec{V}}$$

$$\omega = \frac{d\theta}{dt}$$

Se på ren rulling (dvs uten å gli).

Når legemet har rotert en liten vinkel $d\theta$ omkring CM, har CM flyttet seg en lengde dx som tilsvarer buelengden $R \cdot d\theta$.

Dermed: $\frac{dx}{dt} = \frac{R \cdot d\theta}{dt} \Rightarrow$

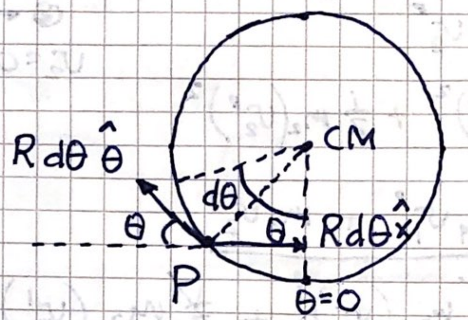
$$\boxed{V = R \cdot \omega}$$ Rullebetingelsen

Som i neste omgang gir $A = \frac{dV}{dt} = R\frac{d\omega}{dt} = R\alpha$;

$\alpha = \dot{\omega}$ = vinkelakselerasjonen

$\boxed{A = R\alpha}$ er også en rullebetingelse

(35) Forflytning $d\vec{r}$ for punkt P på periferien:



Fra figur:

$$\hat{\theta} = -\hat{x}\cos\theta + \hat{y}\sin\theta$$

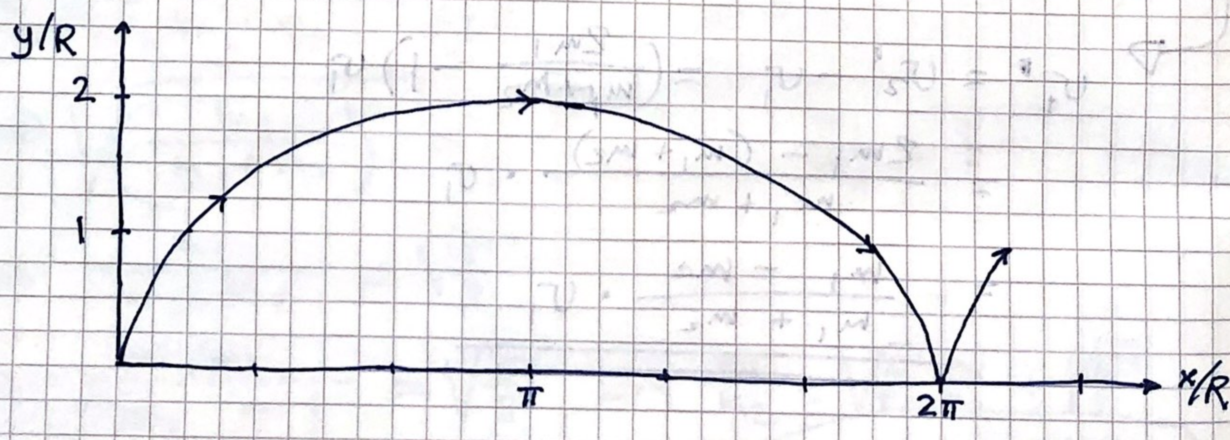
$$d\vec{r} = R\,d\theta\,\hat{x} + R\,d\theta\,\hat{\theta}$$

$$= R\,d\theta\,(1-\cos\theta)\,\hat{x} + R\,d\theta\,\sin\theta\,\hat{y}$$

$\Rightarrow$ Posisjonen $\vec{r}$ til P, med start i $\vec{r} = (x, y) = (0, 0)$ når $\theta = 0$:

$$\vec{r} = \int d\vec{r} = \int_0^\theta \left\{ R\,d\theta(1-\cos\theta)\,\hat{x} + R\,d\theta\,\sin\theta\,\hat{y} \right\}$$

$$= \hat{x}\,R\,(\theta - \sin\theta) + \hat{y}\,R\,(1-\cos\theta)$$

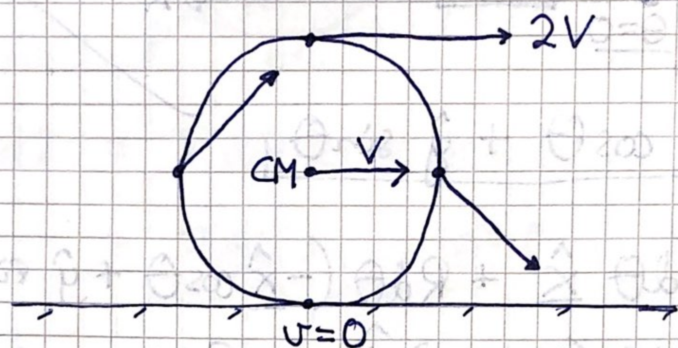


en sykloide

(36) Farten til P:

$$\vec{v} = \frac{d\vec{r}}{dt} = \hat{x}\,(R\dot{\theta} - R\dot{\theta}\cos\theta) + \hat{y}\,R\dot{\theta}\sin\theta$$

$$= \hat{x}\,V(1-\cos\theta) + \hat{y}\,V\sin\theta$$



Når P har kontakt med underlaget, er $v = 0$

$\Rightarrow$ Null effekttap pga friksjon:

$$P_f = \vec{f}\cdot\vec{v} = 0 \quad ; \quad \text{også når } f \neq 0$$

Siden $\omega = V/R$ ved ren rulling, kan vi slå sammen K_{trans} og K_{rot}:

$$K = \frac{1}{2}MV^2 + \frac{1}{2}I_0\omega^2 = \frac{1}{2}MV^2 + \frac{1}{2}\cdot cMR^2\cdot\frac{V^2}{R^2}$$

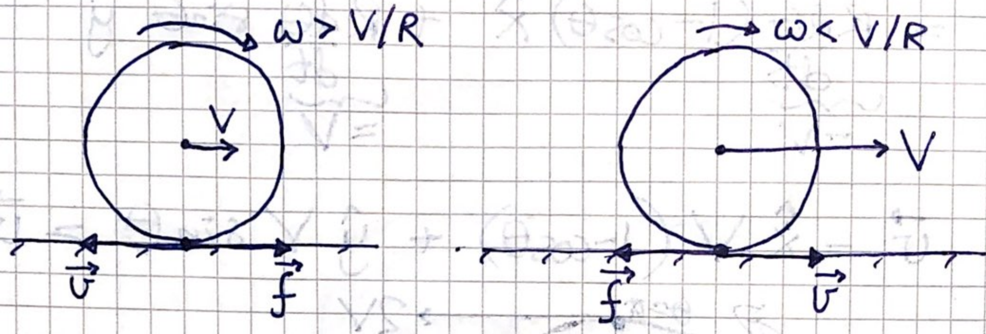
$$= \frac{1}{2}(1+c)MV^2$$

Objekttype	Kompakt kule	Kompakt skive	Kuleskall	Tynn ring
c-verdi	2/5	1/2	2/3	1

H.t 27.09.21

Hvis $\omega \neq V/R$, har kontaktpunktet fart

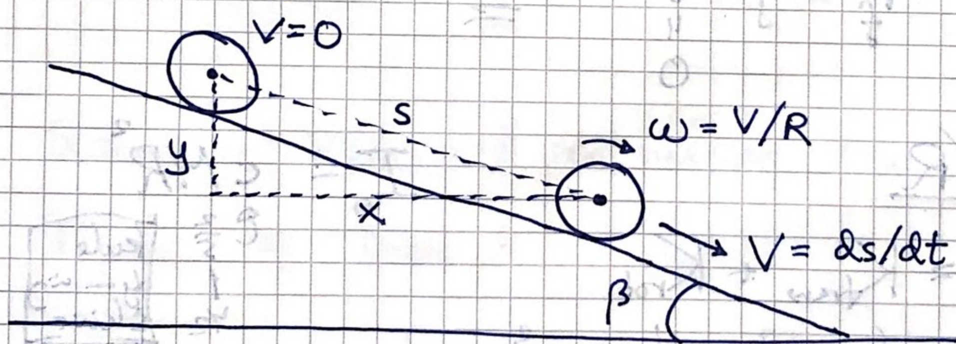
$v = V - \omega R \neq 0$, og legemet slurer (roterer og glir).



$P_f = \vec{f} \cdot \vec{v} < 0$ ($f = \mu_k \cdot N$; kinetisk friksjon)

Legemer taper mekanisk energi $|P_f|$ pr tidsenhet

Ren rulling på skråplan



Finn V, A og f, samt maksimal β som gir ren rulling

Fra sist: Rullebetingelsen: $V = \omega R$

$$K_{rot} = \frac{1}{2} I_0 \omega^2 = \frac{1}{2} \cdot c M R^2 \cdot \frac{V^2}{R^2} = \frac{c}{2} M V^2$$

(38) Energibevarelse: $Mgy = \frac{1}{2}(1+c)MV^2$

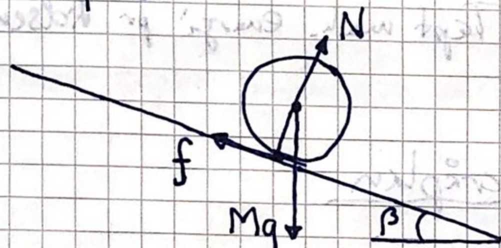
$$\Rightarrow \quad V = \sqrt{\frac{2gy}{1+c}} \qquad (y = s \cdot \sin\beta)$$

Akselerasjon:

$$\underline{A} = \frac{dV}{dt} = \frac{dV}{dy}\frac{dy}{ds}\frac{ds}{dt}$$

$$= \sqrt{\frac{2g}{1+c}} \cdot \frac{1}{2} y^{-1/2} \cdot \sin\beta \cdot V = \underline{\underline{\frac{g\sin\beta}{1+c}}}$$

Krefter:



N1 $\perp$ skråplanet $\Rightarrow$ $\underline{N = Mg\cos\beta}$

N2 $\parallel$ —"— $\Rightarrow$ $Mg\sin\beta - f = MA$

$$\Rightarrow \underline{f} = Mg\sin\beta - M \cdot \frac{g\sin\beta}{1+c}$$

$$= Mg\sin\beta\left(1 - \frac{1}{1+c}\right) = \underline{\underline{\frac{c}{1+c} Mg\sin\beta}}$$

Ren rulling betyr <u>statisk</u> friksjon.

Fra før (30.08.21, s. 11): $f \leq \mu_s \cdot N$

$$\Rightarrow \quad \frac{c}{1+c} Mg\sin\beta \leq \mu_s Mg\cos\beta$$

$$\Rightarrow \quad \underline{\underline{\tan\beta \leq \mu_s \cdot \frac{1+c}{c}}}$$

(39) Plast mot plast og metall mot plast: $\mu_s \approx 0.2 - 0.5$

Eks: Kompakt skive, $c = 1/2$; $\mu_s = 0.2$

$$\tan\beta \leq 0.2 \cdot \frac{1+0.5}{0.5} = 0.6$$

$$\Rightarrow \beta \leq \arctan 0.6 = 31°$$

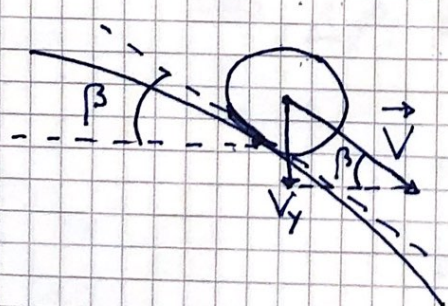
Ren rulling på berg-og-dal-bane

Varierende helningsvinkel β; kan ha rulling både oppover og nedover; banen krummer oppover og nedover.

Energibevarelse gir

$$V = \sqrt{\frac{2gy}{1+c}} \quad \text{og} \quad A_{\parallel} = \frac{g\sin\beta}{1+c}$$

som med skråplan:



$$A_{\parallel} = \frac{dV}{dt} = \sqrt{\frac{2g}{1+c}} \cdot \frac{1}{2} y^{-1/2} \cdot \frac{dy}{dt}$$

$$\frac{dy}{dt} = V_y = V\sin\beta = \sqrt{\frac{2gy}{1+c}}\sin\beta$$

$$\Rightarrow A_{\parallel} = \frac{g\sin\beta}{1+c}$$

N2 $\parallel$ banen: $Mg\sin\beta - f = MA_{\parallel}$

$$\Rightarrow f = \frac{c}{1+c} Mg\sin\beta$$; som skråplan

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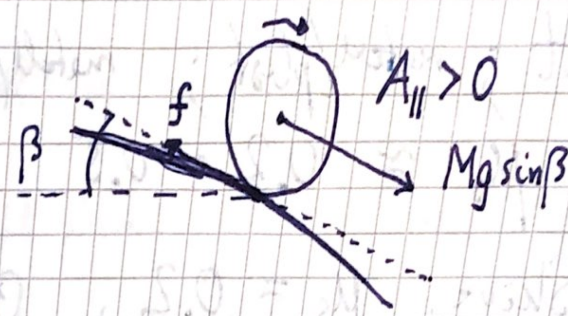
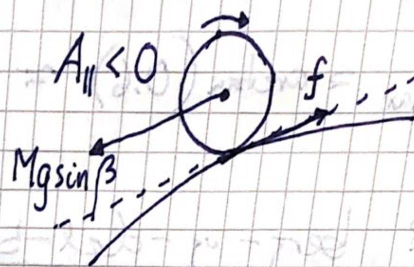
Nedover:

$\beta > 0$



Oppover:

$\beta < 0$

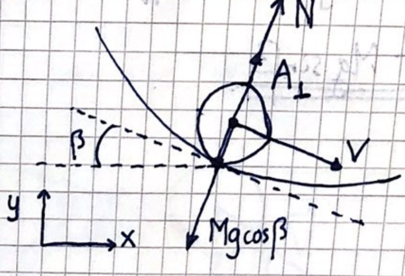


N2 ⊥ banen: $\sum F_{\perp} = MA_{\perp}$; $A_{\perp} = \frac{V^2}{\rho}$

ρ = banens krumningsradius

$= \left[1 + (y')^2\right]^{3/2} / |y''|$

Krumning oppover: $y'' > 0$ (når y øker oppover)



$y' = \frac{dy}{dx} = -\tan\beta$ $(\beta > 0)$

$\sum F_{\perp} = N - Mg\cos\beta$

$A_{\perp} = V^2/\rho > 0$

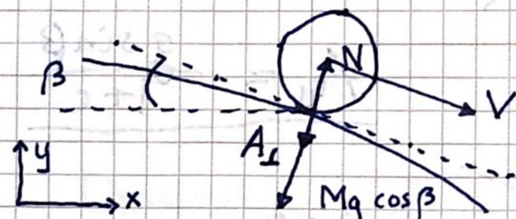
$V^2 = \frac{2gy}{1+c}$; y = høydeendring fra startpos. $(y > 0)$

$N = MA_{\perp} + Mg\cos\beta = MV^2/\rho + Mg\cos\beta$

$= M \cdot \frac{2gy}{1+c} \cdot \frac{y''}{\left[1+(y')^2\right]^{3/2}} + Mg\cos\beta$

(41) Krumning nedover: $y'' < 0$

$y' = -\tan\beta$



$$MA_\perp = Mg\cos\beta - N$$

$$\Rightarrow N = Mg\cos\beta - MA_\perp = Mg\cos\beta - M\frac{V^2}{|\rho|}$$

$$= Mg\cos\beta - M\cdot\frac{2gy}{1+c}\cdot\frac{|y''|}{[1+(y')^2]^{3/2}}$$

For å ha <u>ren rulling</u> må vi også her ha en statisk friksjonskraft

$$f \leq \mu_s \cdot N,$$

dvs $\mu_s \cdot N \geq \frac{c}{1+c} Mg\sin\beta$

Dvs: Normalkraften N fra banen må hele veien være tilstrekkelig stor, slik at ulikheten

$$f/N \leq \mu_s$$

er oppfylt. Vi ser at stor fart V og krumning nedover bidrar til at N kan bli (for?) liten.