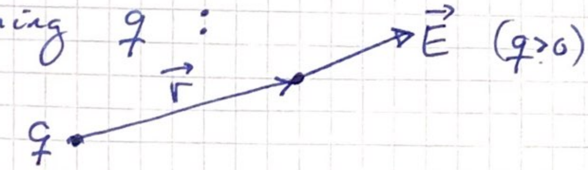
Sist: Coulombs lov: $F_{12} = \dfrac{q_1 q_2}{4\pi\varepsilon_0 r_{12}^2}$; $[q] = C$

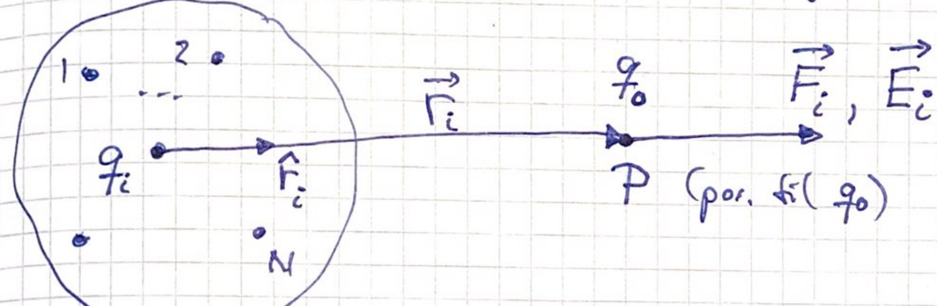
Elektrisk felt, fra ladning q:

$$\vec{E} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r}$$



Superposisjonsprinsippet ("SPP"):

Total kraft, og dermed totalt el. felt, fra flere ladn. er vektorsummen av enkeltbidragene.

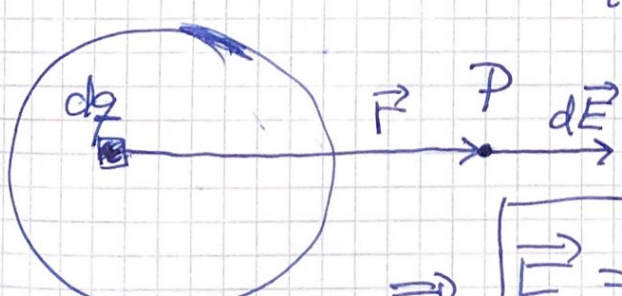


$$\vec{E} = \vec{F}/q_0 = \sum_{i=1}^{N} \frac{q_i \cancel{q_0}\, \hat{r}_i}{4\pi\varepsilon_0 r_i^2 \cancel{q_0}} = \sum_i \underbrace{\frac{q_i \hat{r}_i}{4\pi\varepsilon_0 r_i^2}}_{=\vec{E}_i} = \sum_i \vec{E}_i$$

= el felt i pos P fra ref. ladn. $\{q_1, \dots, q_N\}$

Med kontinuerlig fordeling av (ref.) ladning:

$$q_i \rightarrow dq \quad ; \quad \sum_i \rightarrow \int$$

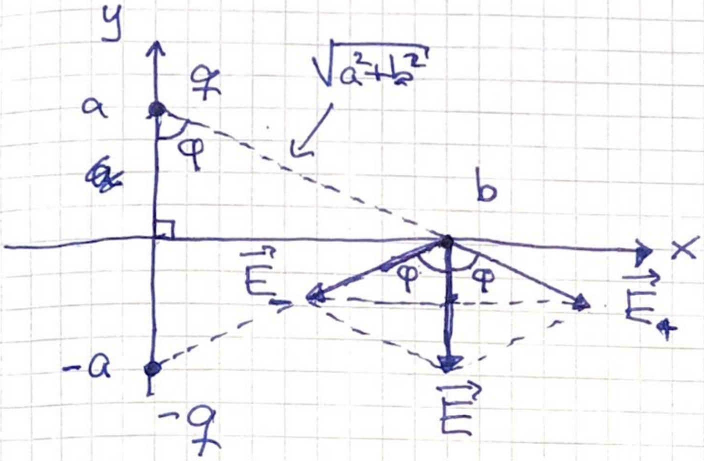


$$d\vec{E} = \frac{dq \cdot \hat{r}}{4\pi\varepsilon_0 r^2}$$

$$\Rightarrow \boxed{\vec{E} = \int d\vec{E} = \int \frac{\hat{r}\, dq}{4\pi\varepsilon_0 r^2}} = \text{el. felt i pos P}$$

Eks 1: Elektrisk dipol

Punktladn. $\pm q$ i $y = \pm a$ $(x=0)$;
Hva er $\vec{E}$ i $x = b$ $(y=0)$?



$$\vec{E} = -E\,\hat{y}$$

$$E = 2\cdot E_+ \cdot \cos\varphi$$

$$E_+ = E_- = \frac{q}{4\pi\varepsilon_0 (a^2+b^2)}$$

$$\cos\varphi = \frac{a}{\sqrt{a^2+b^2}}$$

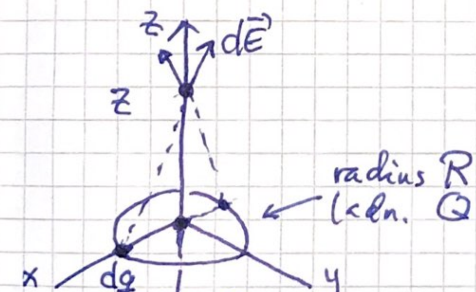
$$\Rightarrow \underline{\underline{\vec{E} = -\hat{y}\cdot\frac{2qa}{4\pi\varepsilon_0 (a^2+b^2)^{3/2}}}}$$

Hvis $b \gg a$: $a^2+b^2 \approx b^2$

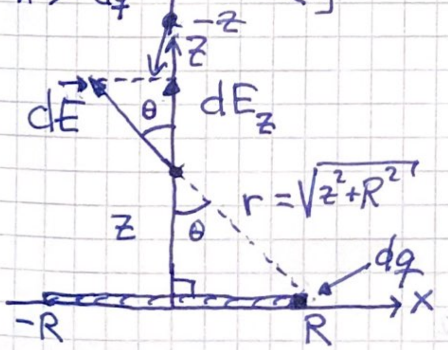
$$\Rightarrow E(b) \approx \frac{2q\cdot a}{4\pi\varepsilon_0 b^3} \sim \frac{1}{b^3}$$

$\Rightarrow$ E går mot 0 "som" $1/b^3$, raskere enn $1/b^2$

Eks 2: $\vec{E}$ på aksen til en jevnt ladet ring



Symmetri $\Rightarrow$ $\vec{E}(z) = E_z(z)\,\hat{z}$
på z-aksen



Bidrag fra dq på x-aksen:

$$dE_z = dE\cdot\cos\theta$$

$$dE = \frac{dq}{4\pi\varepsilon_0 r^2}$$

$$\cos\theta = z/r$$

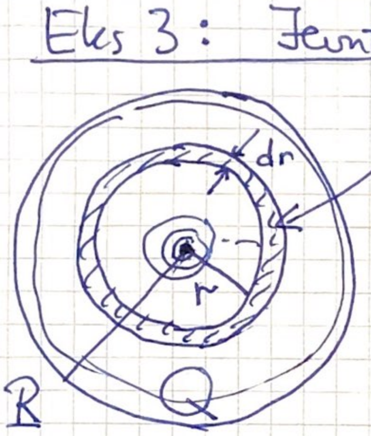
$$E_z(z) = \int dE_z = \int dE \cdot \cos\theta = \int \frac{dq \cdot z}{4\pi\varepsilon_0 (z^2+R^2)\cdot\sqrt{z^2+R^2}}$$

$$= \frac{z\, Q}{4\pi\varepsilon_0 (z^2+R^2)^{3/2}} \qquad \left(\int dq = Q\right)$$

Fornuftig? Rett enhet. $E_z(0) = 0$. $E_z(-z) = -E_z(z)$.

$E_z(z) \overset{z\gg R}{\approx} Q/4\pi\varepsilon_0 z^2$, som pkt.ladn. Q i origo.

Eks 3: Jevnt ladet skive = sum av tynne ringer



Ladn. på tynn ring, radius r, bredde dr:

$$\frac{dq}{Q} = \frac{2\pi r \cdot dr}{\pi R^2} \Rightarrow dq = Q \cdot \frac{2r\,dr}{R^2}$$

Bidrag til E_z fra denne ringen:

$$dE_z = \frac{z \cdot dq}{4\pi\varepsilon_0 (z^2+r^2)^{3/2}} = \frac{z\cdot Q \cdot r\,dr}{2\pi\varepsilon_0 R^2 (r^2+z^2)^{3/2}}$$

$\Rightarrow$ Totalt felt på z-aksen (= symm. aksen)

$$E_z = \int dE_z = \frac{Qz}{2\pi\varepsilon_0 R^2}\int_{r=0}^{R} \frac{r\cdot dr}{(r^2+z^2)^{3/2}}$$

$$= \frac{Qz}{2\pi\varepsilon_0 R^2}\Big|_0^R \left\{-(r^2+z^2)^{-1/2}\right\}$$

$$= \frac{Qz}{2\pi\varepsilon_0 R^2}\left\{\frac{1}{z} - \frac{1}{\sqrt{R^2+z^2}}\right\}$$

$$= \frac{Q}{2\pi\varepsilon_0 R^2}\left\{1 - \frac{\sqrt{z^2}}{\sqrt{z^2+R^2}}\right\}$$

$$= \frac{Q}{2\pi\varepsilon_0 R^2}\left\{1 - \frac{1}{\sqrt{1+R^2/z^2}}\right\}$$