NORGES TEKNISK-
NATURVITENSKAPELIGE UNIVERSITET
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EXAM
MNFFY 103 ELECTRICITY AND MAGNETISM
Wednesday May 21, 2003
9 am - 3 pm
English version

Remedies: C

- K. Rottmann: Matematisk formelsamling
- O. Øgrim and B. E. Lian: Størrelser og enheter i fysikk og teknikk
- Approved calculator, with empty memory, according to list composed by NTNU

Pages 2 - 9: Exercises 1 - 6.
Appendix 1 - 3: Formulas.

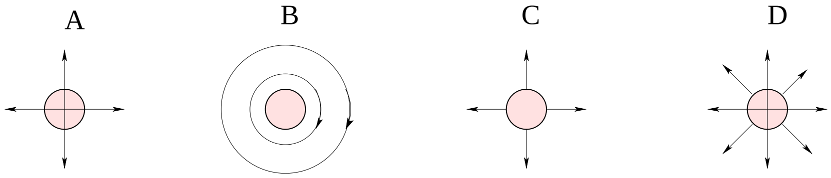
The exam consists of 10 questions (1, 2, 3, 4a, 4b, 5a, 5b, 6a, 6b, 6c) that all count with equal weight in the evaluation. Vectors are given in **boldface** in the text. Unless explicitly stated otherwise, you may assume that the surrounding medium is air (vacuum) with permittivity $\varepsilon_0 = 8.85 \cdot 10^{-12}$ F/m and permeability $\mu_0 = 4\pi \cdot 10^{-7}$ H/m.

The marks may be expected around June 11.

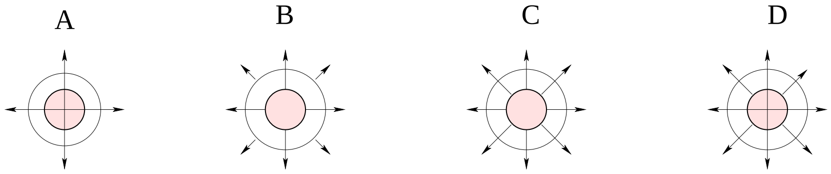
EXERCISE 1

This exercise consists of 5 questions with 4 alternatives given for each question. Only one of them is correct. Answer each question with *one* letter, A, B, C or D. Each correct answer gives 2 points. A wrong answer gives 0 points.

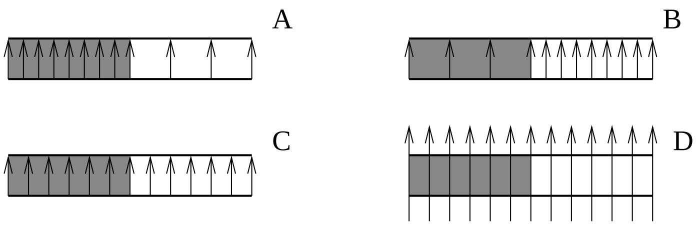
i. The correct figure shows electric field lines in a plane through the centre of a metal sphere (shaded) with net charge $Q > 0$:



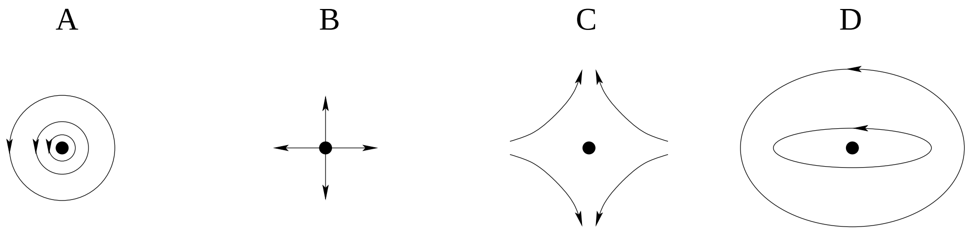
ii. The correct figure shows electric field lines in a plane through the centre of a charged metal sphere (shaded) evenly covered with electrically neutral plastic with permittivity $\varepsilon > \varepsilon_0$. The surrounding medium is air:



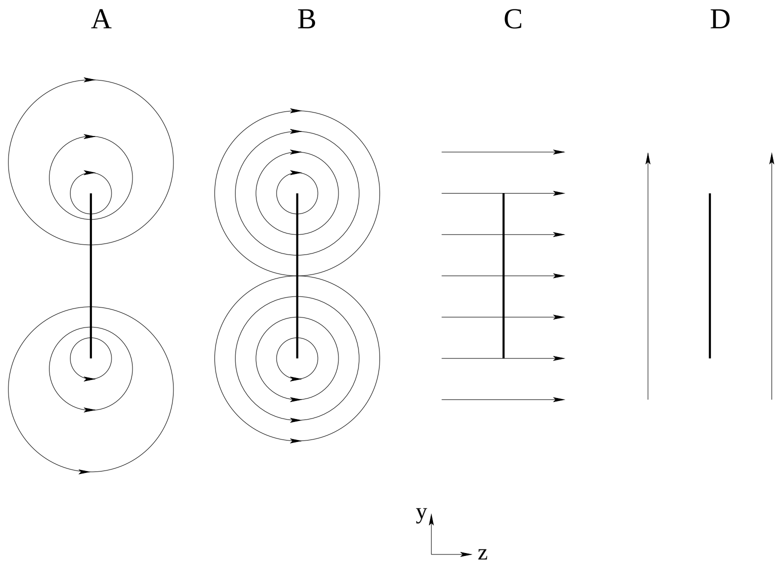
iii. The correct figure shows electric field lines for a parallel plate capacitor half-filled with a dielectric (shaded area). The linear dimension of the plates is large compared to the distance between the plates. The upper metal plate carries a negative charge $-Q$, the lower metal plate carries a positive charge Q:



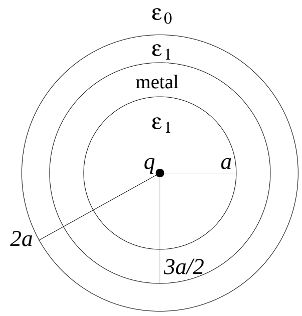
iv. The correct figure shows magnetic field lines for a long, straight current-carrying wire, where the current I comes perpendicularly out of the paper plane:



v. The correct figure shows magnetic field lines in the yz-plane for a circular current loop (thick line) lying in the xy-plane:



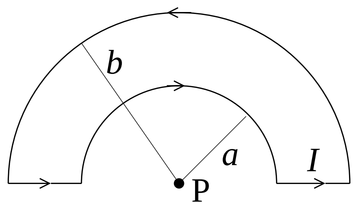
EXERCISE 2



A positive point charge q is located in the centre of a plastic sphere with radius $r = a$. Then follows a metallic spherical shell with thickness $a/2$, and finally a spherical shell of the plastic with thickness $a/2$. Outside, we have air (vacuum). The plastic has permittivity $\varepsilon_1 = 10\varepsilon_0$. Determine the electric field $\boldsymbol{E}$ everywhere. Sketch $E = |\boldsymbol{E}|$ versus r.

Given information: $\oint \boldsymbol{D}\cdot\boldsymbol{dA} = q_{\text{free}}$ $\qquad \boldsymbol{D} = \varepsilon_0\boldsymbol{E} + \boldsymbol{P} = \varepsilon\boldsymbol{E}$

EXERCISE 3



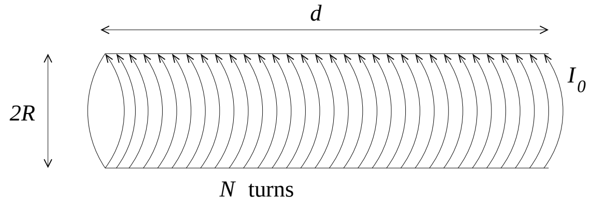
A thin current-carrying wire forms a closed loop that lies partly along the circumference of two concentric circles with radius a and b ($b > a$), respectively, and partly along radial connecting lines, see figure above. The loop carries a current I with direction as shown in the figure. Determine the magnetic field $\boldsymbol{B}$ (magnitude and direction) in the point P, i.e., in the centre of the two circles.

Given information: On the symmetry axis of a circular current loop with radius L, carrying a current I, the magnetic field is

$$B = \frac{\mu_0 I L^2}{2\left(L^2 + d^2\right)^{3/2}}$$

in a distance d from the centre of the loop, along the symmetry axis.

EXERCISE 4



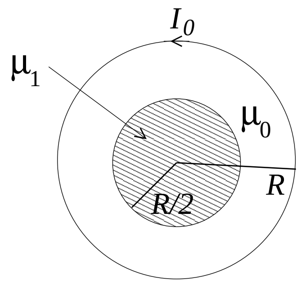
a) Use Ampere's law to show that the magnetic field inside a long (approximately infinite) air-filled solenoid with length d and N evenly and closely spaced turns of a thin wire carrying a current I_0 is

$$B_0 = \frac{\mu_0 N I_0}{d}$$

You may assume that the magnetic field is homogeneous inside the solenoid. Draw a figure to show what kind of "Amperian curve" (i.e., integration curve) you have chosen. Also determine the self-inductance L_0 of such a solenoid when the solenoid has a radius R.

Given information: $\oint \boldsymbol{B} \cdot \boldsymbol{dl} = \mu_0 I$ $\qquad\qquad \Phi = LI$

b) Next, the solenoid is partly filled with a ferromagnetic cylinder with length d and radius $R/2$. The figure below shows a cross section of the solenoid:



The ferromagnetic material has permeability $\mu_1 = \mu_r \mu_0$ (i.e., relative permeability μ_r). When a current I_0 runs in the solenoid wire, the magnetic field B_1 inside the ferromagnetic cylinder ($r < R/2$) becomes different from the field B_0 in the air-filled layer around ($R/2 < r < R$). Explain briefly why this is so. Where is the field largest? What is the self-inductance L_1 of this partly filled solenoid?

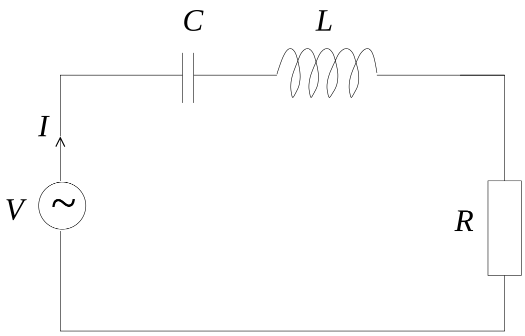
Given information: $\oint \boldsymbol{H} \cdot \boldsymbol{dl} = I_{\text{free}}$ $\qquad\qquad \boldsymbol{B} = \mu_0 \boldsymbol{H} + \mu_0 \boldsymbol{M} = \mu \boldsymbol{H} = \mu_r \mu_0 \boldsymbol{H}$

EXERCISE 5

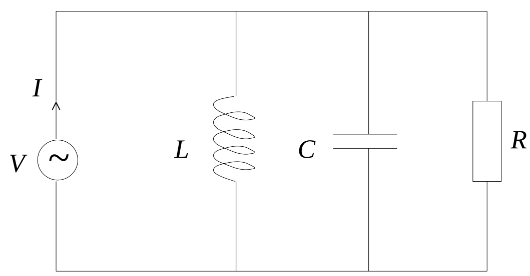
a) Use Kirchhoff's voltage rule to show that the impedance $Z = |Z| \exp(i\alpha) \equiv \{|Z|, \alpha\}$ of a resistance R, capacitance C, and inductance L is $\{R, 0\}$, $\{1/\omega C, -\pi/2\}$, and $\{\omega L, \pi/2\}$, respectively. Use simple circuits, VR-, VC-, and VL-circuit, respectively, with a harmonic voltage source $V = V_0 \cos \omega t$, alternatively with complex notation $V = V_0 \exp(i\omega t)$.

b) You have a resistance $R = 25\ \Omega$, a capacitor $C = 40\ \mu$F, and a solenoid with inductance $L = 250\ \mu$H. Determine the impedance (both absolute value and phase angle) of the following AC circuits when the voltage source is $V = V_0 \cos \omega t$ with angular frequency $1000\ \mathrm{s}^{-1}$.

i.



ii.



Given information:
Complex impedances coupled in series: $Z = \sum_i Z_i$
Complex impedances coupled in parallel: $Z^{-1} = \sum_i Z_i^{-1}$
Meaning of absolute value and phase angle of the complex impedance:
If an AC voltage source $V = V_0 \cos \omega t$ results in an AC current $I = I_0 \cos(\omega t - \alpha)$, the circuit has an impedance with absolute value $|Z| = V_0/I_0$ and phase angle α.
Voltage drop over resistance: RI
capacitance: Q/C
inductance: $L dI/dt$

(you must determine the direction of the voltage drop yourself)

EXERCISE 6

a) Use the assumptions and the information given below to show that the number of electrons in the conduction band (N_c) pr unit volume (V), $n = N_c/V$, for a *pure* semiconductor crystal with bandgap $E_g = E_c - E_v$ has a temperature dependence

$$n = AT^{3/2} \exp\left(-\frac{E_g}{2k_BT}\right)$$

Determine the constant A. (I.e.: Express A in terms of known or given parameters.) Assume that the semiconductor is germanium (Ge) with a bandgap of 0.67 eV. Then, with what factor is n increased upon raising the temperature from 300 to 600 K?

Assumptions:

- Use the free-electron model for the density of states in the conduction band:

$$D_c(E) = \frac{V}{2\hbar^3\pi^2}(2m_c)^{3/2}(E - E_c)^{1/2}$$

  (based on $E(k) = E_c + \hbar^2k^2/2m_c$)

- The probability of a given state with energy E ($E \gg \mu$) being occupied is:

$$f(E) \simeq \exp\left(\frac{\mu - E}{k_BT}\right)$$

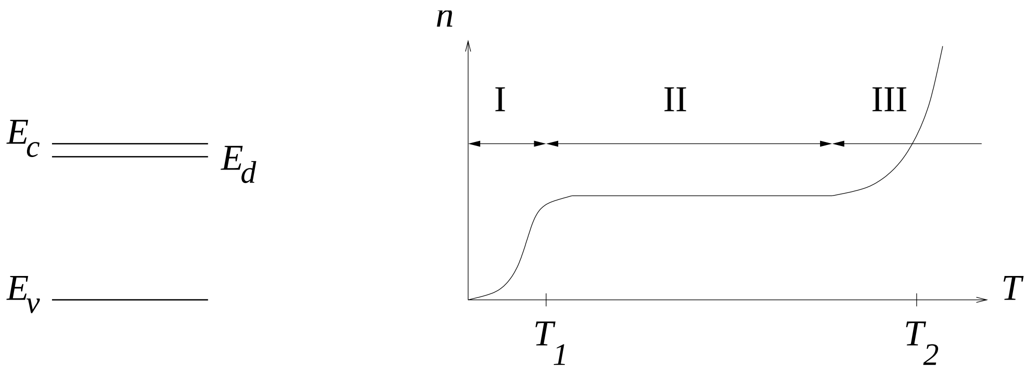
- The chemical potential is located at the centre of the bandgap:

$$\mu = \frac{1}{2}(E_v + E_c)$$

Given information:

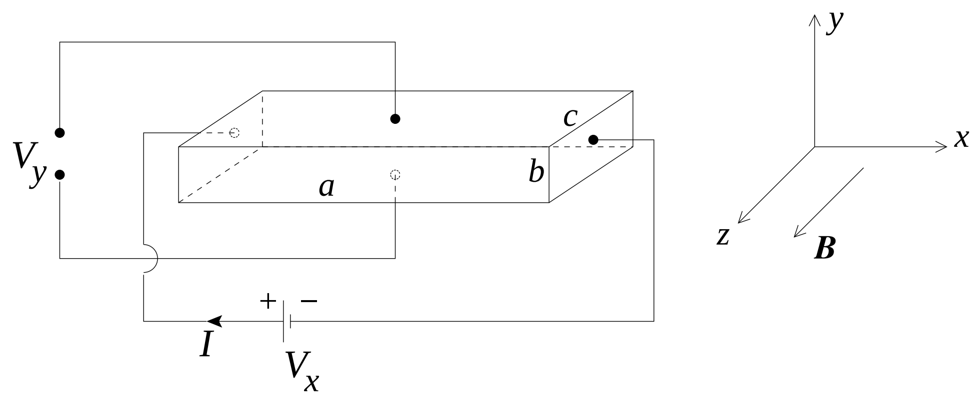
- The number of electrons with energy in the interval $(E, E + dE)$: $dN_c(E) = D_c(E)f(E)dE$
- $\int_0^\infty x^{1/2}e^{-x}dx = \frac{1}{2}\pi^{1/2}$
- Boltzmann's constant: $k_B = 8.617 \cdot 10^{-5}$ eV/K.

b) The left figure illustrates the top of the valence band, E_v, and the bottom of the conduction band, E_c, in the semiconductor germanium (Ge). In an arsenic atom (As) being placed inside a Ge crystal, the electron with maximum energy will have the energy E_d.
The right figure shows the temperature dependence of the concentration of electrons in the conduction band, $n(T)$, for Ge doped with As atoms.



Give a short, qualitative explanation of the behavior of $n(T)$ in the various temperature intervals (I, II, III) when the crystal is heated, starting at $T = 0$. In what way are the two temperatures T_1 and T_2 (orders of magnitude only) related to the energy levels in the left figure?

c) A piece of silicon with length $a = 30$ mm, thickness $b = 3$ mm, and width $c = 5$ mm is doped with so-called donor atoms. At the prevailing temperature, we may with good approximation assume that all donor atoms have donated one electron to the conduction band of silicon. We further assume that the concentration N_D of donor atoms is sufficiently high that we may neglect the "intrinsic" concentration of electrons and holes caused by thermal excitation of electrons from the valence band to the conduction band of silicon. Thus, we only have mobile *electrons* present, with a concentration $n = N_D$.



You are supposed to determine n by means of a Hall experiment, see the figure above. The semiconductor is placed in a homogeneous magnetic field $\boldsymbol{B} = B_z\hat{z}$ (where $\hat{z}$ is a unit vector along the z axis). With an applied voltage V_x across the semiconductor in the x direction, a current I flows. At the same time, you measure a "Hall voltage" V_y across the semiconductor in the y direction.
Derive a relation between the electron concentration n and quantities that are being measured in the Hall experiment. Determine n when $B_z = 0.3$ T, $I = 3.4$ mA, and $V_y = 4.7$ mV.
Suppose that the given direction of I in the figure corresponds to a positive current. Then, in what direction does the magnetic force on the electrons act? What is the direction (polarity) of V_y?

Given information:
The Lorentz force: $\boldsymbol{F} = q(\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B})$
Current density: $\boldsymbol{j} = nq\boldsymbol{v}$
The elementary charge: $e = 1.6 \cdot 10^{-19}$ C

Formulas
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$\int \boldsymbol{dA}$ denotes surface integral and $\int \boldsymbol{dl}$ denotes line integral. $\oint$ denotes integral over closed surface or around closed curve. The validity of the formulas and the meaning of the various symbols are assumed to be known.

Electrostatics and magnetostatics

- Coulomb's law:

$$\boldsymbol{F} = \frac{qq'}{4\pi\varepsilon_0 r^2}\hat{r}$$

- Electric field and potential:

$$\boldsymbol{E} = -\nabla V$$

- Electric potential from point charge:

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

- Electric flux:

$$\phi_E = \int \boldsymbol{E} \cdot \boldsymbol{dA}$$

- Gauss' law for electric field:

$$\varepsilon_0 \oint \boldsymbol{E} \cdot \boldsymbol{dA} = q$$

$$\oint \boldsymbol{D} \cdot \boldsymbol{dA} = q_{\text{fri}}$$

$$\varepsilon_0 \nabla \cdot \boldsymbol{E} = \rho$$

$$\nabla \cdot \boldsymbol{D} = \rho_{\text{fri}}$$

- Electrostatic force is conservative:

$$\oint \boldsymbol{E} \cdot \boldsymbol{dl} = 0$$

- Electric displacement:

$$\boldsymbol{D} = \varepsilon_0 \boldsymbol{E} + \boldsymbol{P} = \varepsilon_r \varepsilon_0 \boldsymbol{E} = \varepsilon \boldsymbol{E}$$

- Electric dipole moment:

$$\boldsymbol{p} = q\boldsymbol{d}$$

- Electric polarization:

$$\boldsymbol{P} = \frac{\Delta \boldsymbol{p}}{\Delta V}$$

- Capacitance:

$$C = \frac{q}{V}$$

- Energy density in electric field:

$$u_E = \frac{1}{2} \boldsymbol{D} \cdot \boldsymbol{E}$$

- Magnetic flux:

$$\phi_B = \int \boldsymbol{B} \cdot \boldsymbol{dA}$$

- Gauss' law for magnetic field:

$$\oint \boldsymbol{B} \cdot \boldsymbol{dA} = 0$$

- Ampère's law:

$$\oint \boldsymbol{B} \cdot \boldsymbol{dl} = \mu_0 I$$

$$\oint \boldsymbol{H} \cdot \boldsymbol{dl} = I_{\text{fri}}$$

- Magnetic field from current-carrying wire:

$$\boldsymbol{B} = \frac{\mu_0}{4\pi} I \int \frac{\boldsymbol{dl} \times \hat{r}}{r^2}$$

- Magnetizing field $\boldsymbol{H}$:

$$\boldsymbol{H} = \frac{1}{\mu_0} \boldsymbol{B} - \boldsymbol{M} = \frac{1}{\mu_r \mu_0} \boldsymbol{B} = \frac{1}{\mu} \boldsymbol{B}$$

- Magnetic moment:

$$\boldsymbol{\mu} = I\boldsymbol{A}$$

- Magnetization:

$$\boldsymbol{M} = \frac{\Delta\boldsymbol{\mu}}{\Delta V}$$

- Magnetic force on straight current-carrying wire:

$$\boldsymbol{F} = I\boldsymbol{L} \times \boldsymbol{B}$$

- Energy density in magnetic field:

$$u_B = \frac{1}{2}\boldsymbol{B} \cdot \boldsymbol{H}$$

Electromagnetic induction

- Faraday–Henry's law:

$$\mathcal{E} = \oint \boldsymbol{E} \cdot \boldsymbol{dl} = -\frac{\partial}{\partial t}\phi_B$$

- Ampère–Maxwell's law:

$$\oint \boldsymbol{B} \cdot \boldsymbol{dl} = \mu_0 \left(I + \varepsilon_0 \frac{d}{dt} \int \boldsymbol{E} \cdot \boldsymbol{dA} \right)$$

- Self inductance:

$$L = \frac{\phi_B}{I}$$

- Mutual inductance:

$$M_{12} = \frac{\phi_1}{I_2} \quad , \quad M_{21} = \frac{\phi_2}{I_1}$$

$$M_{12} = M_{21} = M$$

- Energy density in electromagnetic field:

$$u = \frac{1}{2}\boldsymbol{D} \cdot \boldsymbol{E} + \frac{1}{2}\boldsymbol{B} \cdot \boldsymbol{H}$$