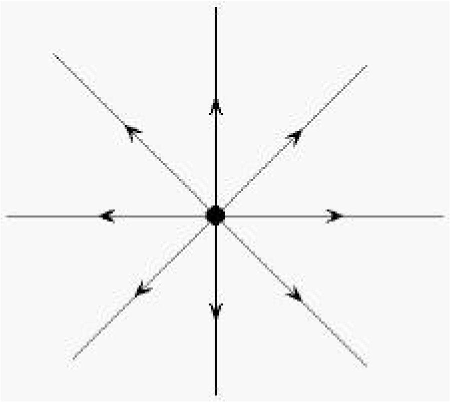
Institutt for fysikk, NTNU
Fag SIF 4012 Elektromagnetisme
Høst 2001

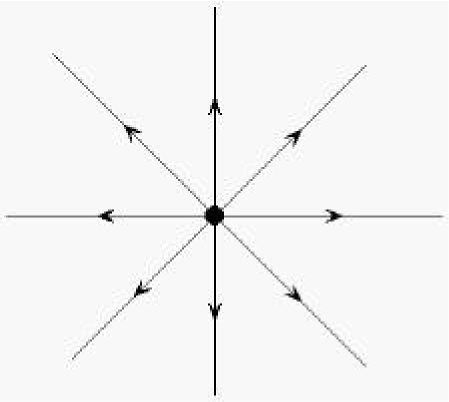
Løsning øving 2

Oppgave 1

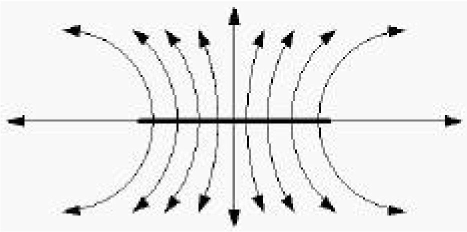
a) Nært:



Langt unna:



b) Nært:



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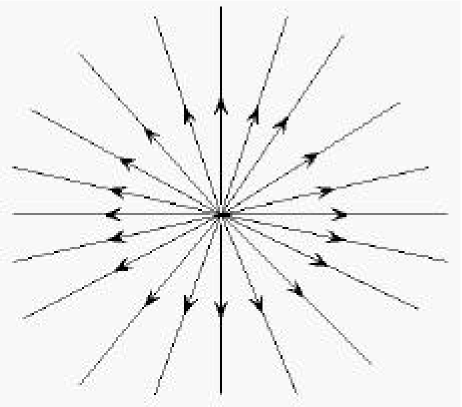
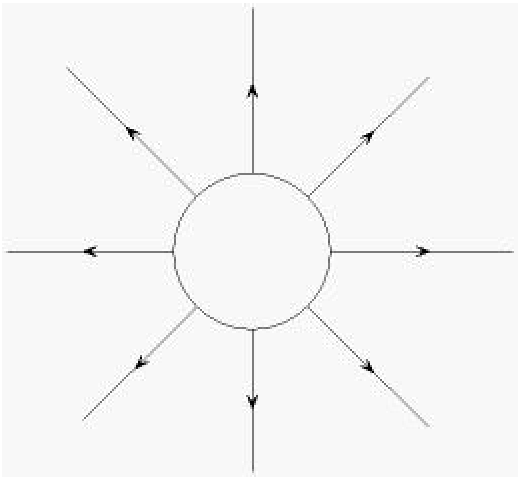


Oppgave 2

a) Nært:



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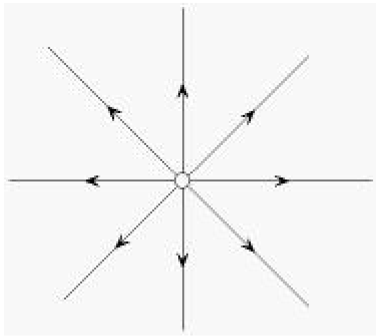
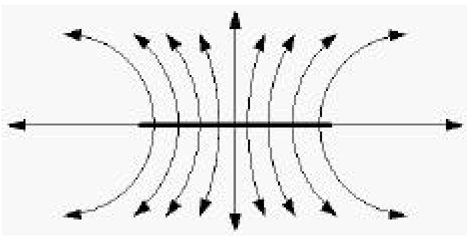
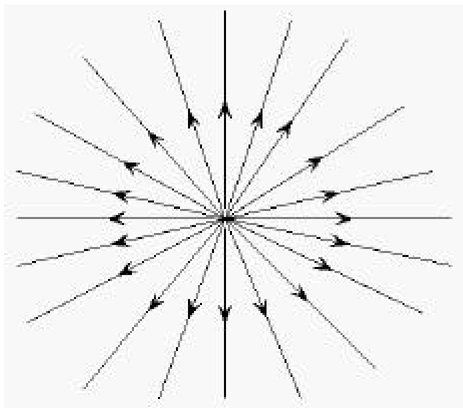


b) Nært:

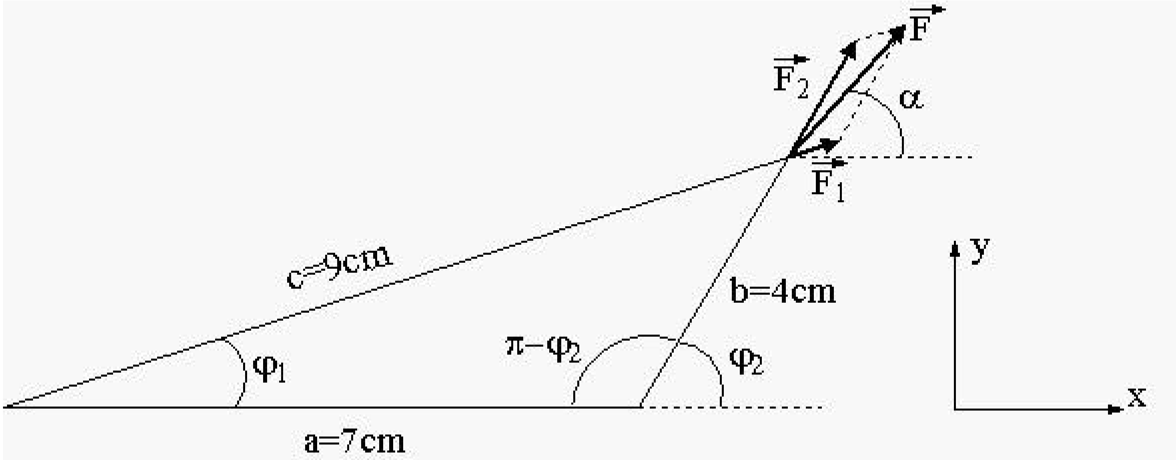


Langt unna:



Legg merke til at både staven og skiva kan tilnærmet betraktes som punktladninger når vi er tilstrekkelig langt unna.

Oppgave 3



Når sidene i trekanten er kjent, bestemmes cosinus til vinklene ved

$$
\begin{aligned}
& b^2 = a^2 + c^2 - 2ac\cos\varphi_1 \\
& \Rightarrow \cos\varphi_1 = \frac{a^2 + c^2 - b^2}{2ac} = 0.9048 \\
& \Rightarrow \varphi_1 = 25.2^\circ \\
& \cos(\pi - \varphi_2) = \frac{a^2 + b^2 - c^2}{2ab} = -0.2857 \\
& \Rightarrow \varphi_2 = (180 - 106.6)^\circ = 73.4^\circ
\end{aligned}
$$

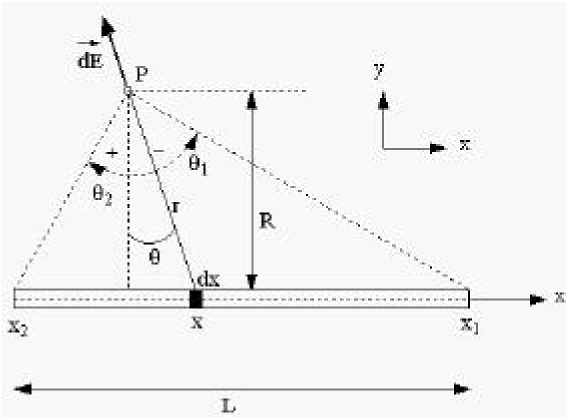
De ulike kraftkomponentene blir dermed (med $K_e = 1/4\pi\varepsilon_0$):

$$
\begin{aligned}
& F_{1x} = K_e \frac{q_3 q_1}{c^2}\cos\varphi_1 = 9.0\cdot 10^9 \cdot \frac{1.5\cdot 10^{-9}\cdot 5.0\cdot 10^{-9}}{\left(9\cdot 10^{-2}\right)^2}\cdot 0.9048\,\mathrm{N} = 7.54\,\mu\mathrm{N} \\
& F_{2x} = K_e \frac{q_3 q_2}{b^2}\cos\varphi_2 = 8.43\,\mu\mathrm{N} \\
& F_x = F_{1x} + F_{2x} = 16.0\,\mu\mathrm{N} \\
& F_{1y} = F_{1x}\frac{\sin\varphi_1}{\cos\varphi_1} = F_{1x}\tan\varphi_1 = 7.54\,\mu\mathrm{N}\cdot\tan\left(25.2^\circ\right) = 3.55\,\mu\mathrm{N} \\
& F_{2y} = F_{2x}\tan\varphi_2 = 28.3\,\mu\mathrm{N} \\
& F_y = F_{1y} + F_{2y} = 31.9\,\mu\mathrm{N} \\
& F = \sqrt{F_x{}^2 + F_y{}^2} = \underline{35.7\,\mu\mathrm{N}}
\end{aligned}
$$

Retningen på totalkraften F er bestemt av

$$
\begin{aligned}
& \tan\alpha = \frac{F_y}{F_x} = \frac{31.9}{16.0} = 1.994 \\
& \Rightarrow \alpha = \underline{63.4^\circ}
\end{aligned}
$$

Oppgave 4



a) Elektrisk felt fra lengdeelement dx:

$$d\vec{E} = A\frac{dx}{r^2}\hat{r}$$

der $A = \lambda/4\pi\varepsilon_0$. Det gir komponentene

$$dE_x = dE\sin\theta$$
$$dE_y = dE\cos\theta$$

med fortegnsvalg som i oppgaven, dvs $\theta < 0$ når $x > 0$. Vi har følgende sammenhenger:

$$R = r\cos\theta \quad \text{eller} \quad r = \frac{R}{\cos\theta} \quad (R = \text{konst.})$$
$$-x = r\sin\theta = R\tan\theta$$
$$-dx = R\frac{d\theta}{\cos^2\theta}$$
$$-\frac{dx}{r^2} = \frac{1}{\mathrm{R}}d\theta$$
$$\int_{x_2}^{x_1}\frac{dx}{r^2} = -\int_{x_1}^{x_2}\frac{dx}{r^2} = \int_{\theta_1}^{\theta_2}\frac{d\theta}{R}$$

Integrasjon gir dermed:

$$E_x = \int dE_x = \frac{A}{R}\int_{\theta_1}^{\theta_2}\sin\theta\, d\theta = \frac{A}{R}\Big|_{\theta_1}^{\theta_2}(-\cos\theta) = \underline{\frac{A}{R}(\cos\theta_1 - \cos\theta_2)}$$

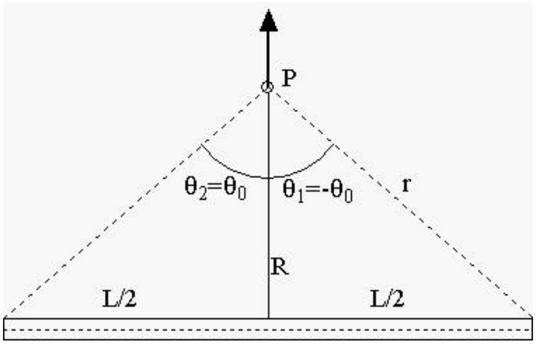
$$E_y = \int dE_y = \frac{A}{R}\int_{\theta_1}^{\theta_2}\cos\theta\, d\theta = \frac{A}{R}\Big|_{\theta_1}^{\theta_2}(\sin\theta) = \underline{\frac{A}{R}(\sin\theta_2 - \sin\theta_1)}$$

(der altså $\theta_1 < 0$ og $\theta_2 > 0$).

b) Langs midtnormalen har en:

$$\sin q_2 = -\sin q_1 = \sin q_0 = \frac{L}{2r}$$

$$\cos q_2 = \cos q_1 = \cos q_0$$

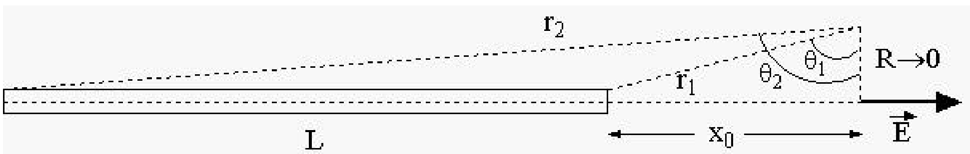


Følgelig finnes langs midtnormalen:

$$E_x = 0$$

$$E_y = \frac{A}{R}\frac{L}{r} = \underline{\frac{lL}{4pe_0 R\sqrt{R^2 + \left(\frac{L}{2}\right)^2}}}$$

c) I stavens forlengelse i avstand x_0 finner en



$$\cos q_1 = \frac{R}{r_1} = \frac{R}{\sqrt{x_0^2 + R^2}} = \frac{R}{x_0\sqrt{1 + \left(\frac{R}{x_0}\right)^2}} = \frac{R}{x_0} + \cdots$$

og tilsvarende

$$\cos q_2 = \frac{R}{L + x_0} + \cdots$$

slik at

$$E_x = \lim_{R \to 0} \frac{A}{R}\left(\cos q_1 - \cos q_2\right) = \frac{A}{R}\left(\frac{R}{x_0} - \frac{R}{L + x_0}\right) = \underline{\frac{AL}{x_0\left(L + x_0\right)}}$$

Videre er så, ved rekkeutvikling ($(R/x_0)^2 \to 0$):

$$\sin q_1 = \frac{x_0}{r_1} = \frac{x_0}{\sqrt{x_0^2 + R^2}} = 1 - \frac{1}{2}\left(\frac{R}{x_0}\right)^2 + \cdots$$

$$\sin q_2 = \frac{x_0 + L}{r_2} = 1 - \frac{1}{2}\left(\frac{R}{L + x_0}\right)^2 + \cdots$$

slik at

$$E_y = \lim_{R \to 0} \frac{A}{R}\left[1 - \frac{1}{2}\left(\frac{R}{x_0}\right)^2 + \cdots - \left(1 - \frac{1}{2}\left(\frac{R}{L + x_0}\right)^2 + \cdots\right)\right] = 0$$

Resultatet $E_y=0$ er vel lett å innse ved direkte betraktning. Videre, og som nevnt i oppgaveteksten, kan E_x enkelt bestemmes ved direkte integrasjon:

$$E_x = A\int_{-L}^{0} \frac{dx}{(x_0 - x)^2} = A \Big|_{-L}^{0} \frac{1}{x_0 - x} = A\left(\frac{1}{x_0} - \frac{1}{x_0 + L}\right) = \underline{\frac{AL}{x_0(x_0 + L)}}$$

(når origo legges i høyre ende av staven).