• Lagranges ligninger (holonomt system):

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_j} - \frac{\partial T}{\partial q_j} = Q_j$$

Kons. system $\Rightarrow Q_j = -\partial V/\partial q_j$

$$\Rightarrow \quad \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_j} - \frac{\partial L}{\partial q_j} = 0 \quad ; \quad L = T - V = L(q, \dot{q}, t)$$

L ikke entydig: $L' = L + \frac{dF(q,t)}{dt}$ $\Rightarrow$ samme beveg. lign.

• generaliserte potensialer:

$$Q_j = -\frac{\partial U}{\partial q_j} + \frac{d}{dt}\frac{\partial U}{\partial \dot{q}_j} \quad ; \quad L = T - U \; ; \quad U = U(q, \dot{q})$$

Eks: E.m. potensial $U = q\phi - q\vec{A}\cdot\vec{v} = U(x_i, v_i)$

ladning!

• friksjon:

$$F_{fx} = -k_x v_x = -\frac{\partial}{\partial v_x}\left(\frac{1}{2}k_x v_x^2\right)$$

$$\vec{F}_f = -\nabla_v \mathcal{F} \quad ; \quad \mathcal{F} = \frac{1}{2}\sum_i \left(k_x v_{ix}^2 + k_y v_{iy}^2 + k_z v_{iz}^2\right)$$

$$Q_j = -\frac{\partial \mathcal{F}}{\partial \dot{q}_j}$$

$$\Rightarrow \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} + \frac{\partial \mathcal{F}}{\partial \dot{q}_i} = 0$$